Fourier-Transform-Domain Solution of the Directive Beam Scattering by a Dielectric Slab with Account of the Guided Waves

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Abstract

Considered is a lossless dielectric slab excited by a two-dimensional E-polarized complex source point (CSP) beam. This constitutes an accurate and experimentally realizable model for estimating the power transferred from a finite directive source into the slab's guided waves. The guided-waves fields as well as the space-wave (scattered) fields are analytically determined by employing a Fourier-transform-domain approach. The guided-modes powers and efficiencies and the space-wave powers are calculated as functions of problem's parameters. Some preliminary numerical results on the variations of these powers are presented.

1. Introduction

The fields generated by CSP beams are exact solutions of the Helmholtz and Maxwell equations and additionally satisfy the radiation condition at infinity, unlike the Gaussian beam fields. The CSP beams were proposed in [1, 2] and used as incident fields for the flat interface scattering in [3, 4] and for the reflector and lens antenna configurations in [5-10]. CSP field absorption by a lossy grounded dielectric slab was studied in [11,12]. A CSP beam can model accurately a directive incident field placed at a finite distance from the structure under examination; such a consideration corresponds more closely to an actual experimental situation.

Excitation of a lossless slab by a directive CSP beam placed at a finite distance from it can serve as a realistic model for estimating the power transferred from the source into the slab's guided waves. To this end, in this work we investigate semi-analytically the electromagnetic scattering problem associated with the CSP excitation of a lossless dielectric slab and predict accurately the power captured by the modes and the power radiated to the outer space. Representative numerical results are presented, corresponding to the variations of the guided-waves powers versus some of the problem's parameters. Certain preliminary results for the mathematical analysis of this problem were reported in [13].

The $\exp(+i\omega t)$ time dependence is assumed and suppressed throughout, where ω is the angular frequency.

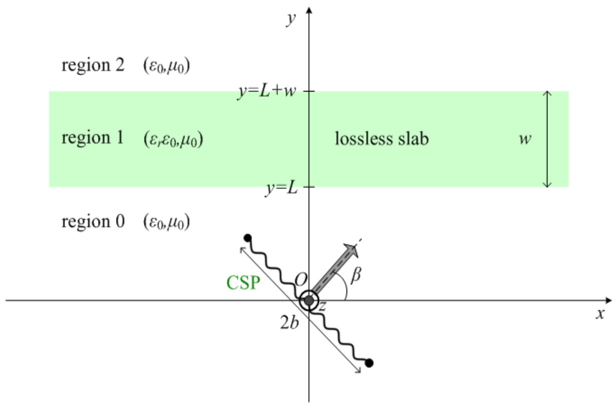

Figure 1. Geometrical configuration of the CSP beam illuminating a lossless dielectric slab. The beam aperture is shown by a wavy line and has width b and orientation angle β.

2. Analytical Considerations

A lossless dielectric slab of thickness w and relative real dielectric permittivity ε_r is excited by a CSP beam (see Fig. 1) with its complex coordinates given by

$$\mathbf{r}_{CS} = (x_{CS}, y_{CS}) = \mathrm{i}\,\mathbf{b}, \quad \mathbf{b} = -b(\cos\beta, \sin\beta), \qquad (1)$$

where the real parameters b and β are, respectively, the aperture width and the orientation angle of a horn antenna simulated by a CSP [1-6]. The distance between the center (0,0) of the CSP aperture and the lower boundary of the slab is denoted by L.

The z-component of the primary electric field, radiated by the CSP in the free space, is given by

$$E^{\mathrm{pr}}(\mathbf{r}, \mathbf{r}_{CS}) = -\tfrac{\mathrm{i}}{4} H_0^{(2)}\left(k_0 \,|\, \mathbf{r} - \mathbf{r}_{CS} \,|\right), \qquad (2)$$

where $H_0^{(2)}$ denotes the second-kind cylindrical Hankel function of order 0, and $k_0=\omega/c$ is the free-space wavenumber (with c for the light velocity).

The interaction of the primary CSP field with the dielectric slab generates in all regions of the problem the secondary field E^{sec}, which satisfies the homogeneous Helmholtz equation with the wavenumbers k_0 and

$k_1=k_0\varepsilon_r^{1/2}$ in the vacuum, and the slab, respectively, as well as the transmission boundary conditions at slab's boundaries. The radiation condition for E^{sec} has to be modified with respect to the free-space radiation condition due to the existence of guided waves in the lossless slab. Following [14], we demand that far from the origin function E^{sec} is the sum of an outgoing cylindrical wave off the slab domain and a finite sum of non-attenuating guided waves both inside and outside of that domain,

$$E^{\text{sec}}(\mathbf{r},\mathbf{r}_{CS}) \sim W(y)\sqrt{\frac{2\pi}{k_0\rho}}e^{-\text{i}\left(k_0\rho-\frac{\pi}{4}\right)}\Phi^{\text{sec}\pm}(\varphi) + \sum_{q=0}^{Q} A_q^{\pm}V_q(y)e^{\mp\text{i}\beta_q x},\ \pm x>0,\quad \rho\to\infty, \tag{3}$$

where $W(y)=1$ for $y<L$ or $y>L+w$ and 0 otherwise, $\Phi^{\text{sec}\pm}$ are the far-field patterns above and below the slab, respectively, while $A_q^{\pm}$ denote the denote the amplitudes of the guided waves propagating along the slab, β_q being the propagation constants and $V_q(y)$ the cross-sectional fields of these waves.

The unknown secondary field has the Fourier-integral expression

$$E^{\text{sec}}(\mathbf{r},\mathbf{r}_{CS}) = \tfrac{1}{4\pi}\int_{-\infty}^{+\infty} e^{-\text{i}\lambda(x-x_{CS})}\gamma(\lambda,y;y_{CS})\,\text{d}\lambda, \tag{4}$$

where for $y\le L$, $L\le y\le L+w$, and $y\ge L+w$

$$\gamma(\lambda,y,y_{CS}) = \begin{cases} A_1\exp[g_0(\lambda)(y-L)], \\ A_2\cosh[g_1(\lambda)(y-L-\frac{w}{2})]+A_3\sinh[g_1(\lambda)(y-L-\frac{w}{2})], \\ A_4\exp[-g_0(\lambda)(y-L-w)], \end{cases} \tag{5}$$

and $g_p(\lambda)=(\lambda^2-k_p^2)^{1/2}$ for p=0,1. The coefficients $A_j=A_j(\lambda)$, j=1,…,4 are determined explicitly by imposing the transmission boundary conditions at $y=L$ and $y=L+w$ [13]. The transcendental equation determining the poles of the integrand function γ is found to be

$$\Delta(\lambda)=\Delta^e(\lambda)\Delta^o(\lambda), \tag{6}$$

where

$$\Delta^e(\lambda)=g_0(\lambda)\cosh[g_1(\lambda)\tfrac{w}{2}]+g_1(\lambda)\sinh[g_1(\lambda)\tfrac{w}{2}]=0,$$
$$\Delta^o(\lambda)=g_0(\lambda)\sinh[g_1(\lambda)\tfrac{w}{2}]+g_1(\lambda)\cosh[g_1(\lambda)\tfrac{w}{2}]=0, \tag{7}$$

with e and o referring to the even and odd waves. The roots of $\Delta(\lambda)$, are denoted by $\lambda=\pm\beta_q$ (q=0,1,2,…,Q). Real-valued roots β_q are finite in number, located between k_0 and k_1 and are the propagation constants of the even and odd natural waves of the lossless slab waveguide.

3. Space-Wave Fields and Powers

The total far-field patterns are determined by employing the method of steepest descent. In the lower half plane (reflection zone), we obtain the final result

$$\Phi^{\text{tot}-}(\varphi)=\Phi^{\text{pr}}(\varphi)+\Phi^{\text{sec}-}(\varphi),\quad \pi<\varphi<2\pi, \tag{8}$$

where

$$\Phi^{\text{sec}-}(\varphi)=\tfrac{-\text{i}}{4\pi}(k_1^2-k_0^2)\frac{\sinh[g_1(k_0\cos\varphi)w]}{2\Delta(k_0\cos\varphi)}e^{k_0 b\cos(\varphi+\beta)+2\text{i}k_0 L\sin\varphi} \tag{9}$$

while

$$\Phi^{\text{pr}}(\varphi)=-(\text{i}/4\pi)e^{k_0 b\cos(\varphi+\beta)},\quad 0\le\varphi\le 2\pi, \tag{10}$$

is the far-field pattern of the primary electric field (2).

Moreover, in the upper half plane (transmission zone), we get

$$\Phi^{\text{sec}+}(\varphi)=\tfrac{k_0}{4\pi}\frac{g_1(k_0\cos\varphi)}{\Delta(k_0\cos\varphi)}\sin\varphi\, e^{k_0 b\cos(\varphi-\beta)+\text{i}k_0 w\sin\varphi},\ 0<\varphi<\pi, \tag{11}$$

Next, we calculate the powers lost due to scattering in the upper and lower half planes. These powers are, respectively, defined as the electromagnetic far-field powers exiting two semi-circles of large radius. The total powers of the space waves in the upper and lower half planes are found to be

$$P^{\text{rad}\pm}=\frac{\pi}{k_0 Z_0}\int_0^{\pm\pi}|\Phi^{\text{tot}\pm}(\varphi)|^2\,\text{d}\varphi. \tag{12}$$

4. Guided-Waves Fields and Powers

By employing the contour integration technique in the complex λ-plane [13], we find that the secondary electric fields expressions $E^{\text{sec}\pm}$ for waves traveling parallel to the $\pm x$ directions in region #0 (i.e. for $y\le L$) take the form

$$E^{\text{sec}\pm}(x,y;b,\beta)=\sum_{q=0}^{Q^e} A_{q,e}^{\pm}V_q^e(y)e^{\mp\text{i}\beta_q^e x}+\sum_{q=1}^{Q^o} A_{q,o}^{\pm}V_q^o(y)e^{\mp\text{i}\beta_q^o x}, \tag{13}$$

where

$$A_{q,e}^{\pm}=\frac{\text{i}}{2}\frac{(a_1(\beta_q^e))^3}{\beta_q^e(k_0^2-k_1^2)(1+g_0(\beta_q^e)\frac{w}{2})}\times \frac{\tan(a_1(\beta_q^e)\frac{w}{2})}{\cos(a_1(\beta_q^e)\frac{w}{2})}e^{\pm\beta_q^e b\cos\beta}e^{-\text{i}g_0(\beta_q^e)b\sin\beta}e^{-g_0(\beta_q^e)L}, \tag{14}$$

$$A_{q,o}^{\pm} = \frac{\mathrm{i}}{2}\frac{(a_1(\beta_q^o))^3}{\beta_q^o(k_0^2-k_1^2)(1+g_0(\beta_q^o)\frac{w}{2})}\times$$
$$\frac{\cot(a_1(\beta_q^o)\frac{w}{2})}{\sin(a_1(\beta_q^o)\frac{w}{2})}e^{\pm\beta_q^o b\cos\beta}e^{-\mathrm{i}g_0(\beta_q^o)b\sin\beta}e^{-g_0(\beta_q^o)L},$$

with the cross-sectional fields of the eigenwaves given by

$$V_q^e(y)=\begin{cases}\cos(a_1(\beta_q^e)\frac{w}{2})e^{-g_0(\beta_q^e)(y-(L+w))}, & y>L+w\\ \cos(a_1(\beta_q^e)(y-L-\frac{w}{2})), & L<y<L+w\\ \cos(a_1(\beta_q^e)\frac{w}{2})e^{g_0(\beta_q^e)(y-L)}, & y<L\end{cases} \quad (15)$$

$$V_q^o(y)=\begin{cases}\sin(a_1(\beta_q^e)\frac{w}{2})e^{-g_0(\beta_q^e)(y-(L+w))}, & y>L+w\\ \sin(a_1(\beta_q^e)(y-L-\frac{w}{2})), & L<y<L+w.\\ -\sin(a_1(\beta_q^e)\frac{w}{2})e^{g_0(\beta_q^e)(y-L)}, & y<L\end{cases}$$

The powers $P^{\mathrm{gw}+}$ and $P^{\mathrm{gw}-}$ of the guided waves traveling to the $+x$ and $-x$ directions, respectively are obtained as

$$P^{\mathrm{gw}\pm}=\frac{1}{2k_0Z_0}\left(\sum_{q=0}^{Q^e}(N_q^e)^2\,|\,A_{q,e}^{\pm}\,|^2+\sum_{q=1}^{Q^o}(N_q^o)^2\,|\,A_{q,o}^{\pm}\,|^2\right), \quad (16)$$

where Z_0 the vacuum impedance, and

$$(N_q^e)^2=\beta_q^e\int_{-\infty}^{+\infty}[V_q^e(y)]^2\,\mathrm{d}y$$
$$(N_q^o)^2=\beta_q^o\int_{-\infty}^{+\infty}[V_q^o(y)]^2\,\mathrm{d}y \quad (17)$$

are the norms of the guided waves, which are used to normalize the guided-wave powers.

5. Numerical Results

For the numerical investigation of the excitation of the guided waves of the lossless slab by means of the CSP beam, we need a reference value of the power. As such a quantity, we consider the power radiated by an isolated CSP feed in the free space, given by [11]

$$P^{\mathrm{pr}}=\frac{1}{8k_0Z_0}I_0(2k_0b)\,, \quad (18)$$

where I_0 denotes the zero-order modified Bessel function.

Some representative variations of the normalized guided-waves powers $P^{\mathrm{gw}+}/P^{\mathrm{pr}}$ and $P^{\mathrm{gw}-}/P^{\mathrm{pr}}$ (as computed by Eq. (16)) versus the CSP orientation angle β, are depicted in Fig. 2. These correspond to a single-mode slab with k_0w=1.5 and ε_r=5. Three cases are considered for the CSP beam's aperture width: k_0b=0.1, 1, and 10.

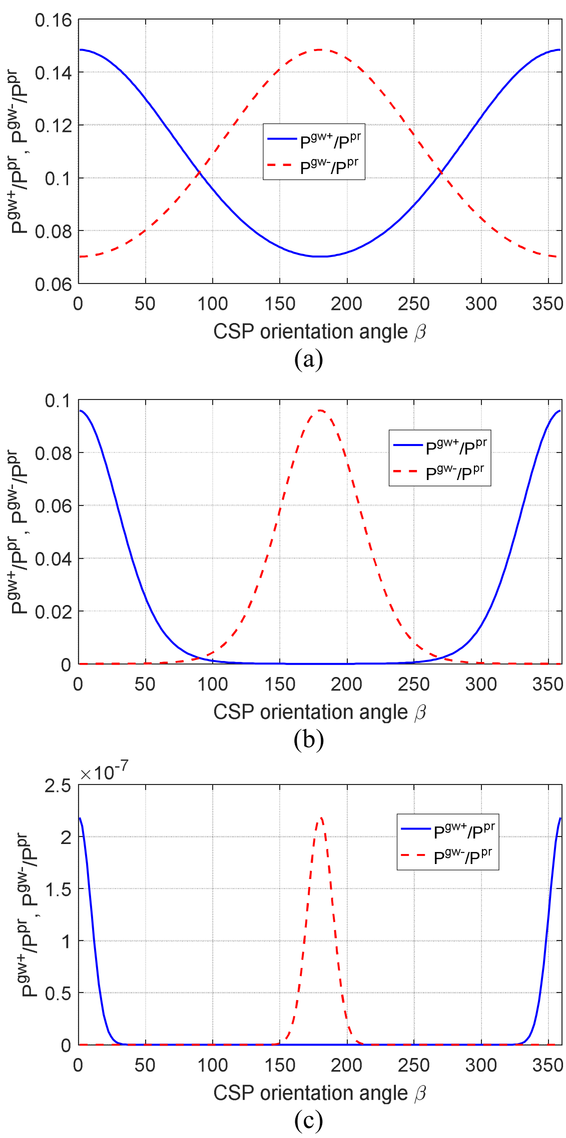

Figure 2. Normalized guided-waves powers $P^{\mathrm{gw}+}/P^{\mathrm{pr}}$ and $P^{\mathrm{gw}-}/P^{\mathrm{pr}}$, corresponding, respectively, to the guided waves traveling parallel to the $+x$ and $-x$ directions, as functions of the CSP beam's orientation angle β for (a) k_0b=0.1, (b) k_0b=1, and (c) k_0b=10, with k_0w=1.5, ε_r=5, and L=1.05b.

In each case the distance of the CSP beam's center to the lower boundary of the slab is L=1.05b.

It can be expected that the grazing incidence of the beam (namely β=0 and π) can be favorable for the excitation of the guided waves. Indeed the curves of $P^{\mathrm{gw}+}/P^{\mathrm{pr}}$ and $P^{\mathrm{gw}-}/P^{\mathrm{pr}}$ have only one maximum either at β=0 or at β=π. Hence, the guided waves can indeed be excited and in fact most efficiently if the CSP beam is in the grazing incidence, although the more collimated the beam the smaller the power of the guided wave. Particularly, at the grazing incidence, only one guided wave is excited with reasonable amplitude, traveling in the same direction where the beam is "looking," while the amplitude of the oppositely traveling wave remains far smaller.

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