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Exceptional Points of Degeneracy in Relation to Singular and Branch Points in the Complex-Frequency Plane

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In recent years there is a considerable interest in exotic phenomena observed in parity-time (PT) quantum, photonic, optical, and metamaterial systems, among others, which possess exceptional points of degeneracy (EPD) of non-Hermitian Hamiltonians. EPDs are the property of a system at which simultaneous eigenvalue and eigenvector degeneracies occur in parameter space. In a system whose evolution is described with an analytic matrix-valued function in linear algebra, EPDs are associated with a Jordan block corresponding to deficient (incomplete) eigenfunctions due to the presence of generalized (associated) eigenvectors at the EPD. It has been observed that in a configuration space wherein multiple branches of spectra connect, the EPDs define branch points (BP) separating different solutions in the space of control variables.

In our previous work, which is ultimately related to the concept of EPD, we have studied various modal interactions on printed-circuit transmission lines and guided-wave structures in connection to singular points of a dispersion function. We have shown that the singular points are associated with the fold-point (FP) bifurcation from bifurcation theory, which in turn define the complex frequency-plane branch points leading to modal degeneracies and modal exchange. In this work, we connect our previous theory of FP-BP and EPD and show that the same equations which model FP-BPs also govern the EPDs for the case of second-order degeneracies. We apply our results to PT-symmetric coupled systems recently considered in the literature, and we consider a generalized case where EPDs occur in complex-frequency space. We also demonstrate that the local form of the dispersion function in the vicinity of FP-BP based on the Taylor series expansion is consistent with the expansion of dispersion function in the vicinity of EPD using a Puiseux series. We show that for the case of real valued EPDs a characteristic dispersion behavior in the neighborhood of EPDs corresponds to the fold bifurcation of the dispersion function. To validate the connection between the concepts of FP-BP and EPD several numerical examples have been studied, including the PT-symmetric coupled transmission line with loss and gain, PT-symmetric 2D parallel plates, and PT-symmetric 2D parallel plates filled with a nonlocal material such as a nonlocal wire medium.