



## Analysis of Pulse Responses from Conducting Strips with Air Layer in Dispersion Media

Ryosuke Ozaki<sup>(1)</sup> and Tsuneki Yamasaki<sup>(1)</sup>  
 (1) College of Science and Technology, Nihon University

### Abstract

In recent papers, we have analyzed the pulse response from periodic perfect conducting strips with reflected plate in two dispersion media by using a combination of fast inversion of Laplace transform method and point matching method, and investigated the influence of both dispersion media and conducting strips.

In this paper, we analyzed the pulse reflection response from periodically conducting strips with air region in dispersion media, and also investigated the influence of number of periodically conducting strips with reflected plate.

### 1. Introduction

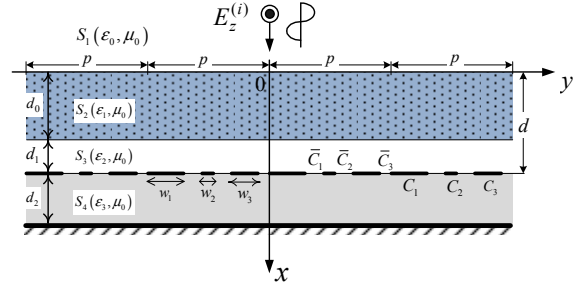
The inverse scattering problem of electromagnetic waves is known as technologies to estimate the target object from scattered waves, and is of interest for imaging technology and development for remote sensing [1, 2]. Numerical methods of them are analyzed by Integration Equation Method (IEM) or Singularity Expansion Method (SEM), and so on [1]. In recent years, the FDTD method is widely employed by the performance enhancement of the computer [3]. Thus, we are required to examine without destroying the target object buried in the subsurface structures. The ground penetrating radar of another radar technology is well known as technology which can investigate the geometry in underground structures [4].

In recent papers, we have analyzed the pulse response from periodic perfect conducting strips with reflected plate in two dispersion media by using a combination of fast inversion of Laplace transform method and point matching method, and also investigated the influence of both dispersion media and conducting strips[5-7].

In this paper, we analyzed the pulse reflection response from periodically conducting strips with air region in dispersion media, and also investigated the influence of number of periodically conducting strips with reflected plate.

### 2. Method of analysis

We consider the structure for periodically conducting strips with air layer in dispersion medium as shown in Fig.1. The structure is uniform in the  $z$ -direction, and is periodic length  $p$  in the  $y$ -direction. The dielectric constant of each regions  $S_1 \sim S_4$  are  $\epsilon_0$ ,  $\epsilon_1 = \epsilon_1(s)$ ,



**Figure 1** Structure and coordinate system of dispersion media with conducting strips and air layer.

$\epsilon_2 = \epsilon_0$ ,  $\epsilon_3 = \epsilon_2(s)$ , respectively. The permeability is assumed to be  $\mu_0$  in all regions. The conducting strips is embedded at  $x = d (= d_0 + d_1)$ , and width of conductor is defined by  $w_1$ ,  $w_2$ , and  $w_3$ .

Next, we formulate the case of normal incidence of electric field in complex frequency domain. And here, time factor of electromagnetic fields is  $\exp(st)$  and suppressed throughout this paper. In the regions  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ , the electromagnetic fields are expressed as

$$E_z^{(1)}(x, y) = E_0^{(1)} e^{-k_1 x} + \sum_{n=-N_1}^{N_1} R_n e^{k_1^{(n)} x - j 2 n \pi y / p}, \quad (1)$$

$$E_z^{(2)}(x, y) = \sum_{n=-N_1}^{N_1} [A_n^{(1)} e^{-k_2^{(n)} x} + B_n^{(1)} e^{k_2^{(n)} x}] e^{-j 2 n \pi y / p}, \quad (2)$$

$$E_z^{(3)}(x, y) = \sum_{n=-N_1}^{N_1} [A_n^{(2)} e^{-k_3^{(n)} x} + B_n^{(2)} e^{k_3^{(n)} x}] e^{-j 2 n \pi y / p}, \quad (3)$$

$$E_z^{(4)}(x, y) = \sum_{n=-N_1}^{N_1} [A_n^{(3)} e^{-k_4^{(n)} x} + B_n^{(3)} e^{k_4^{(n)} x}] e^{-j 2 n \pi y / p}, \quad (4)$$

$$H_y^{(t)}(x, y) := \frac{1}{\mu_0 s} \frac{\partial E_z^{(t)}(x, y)}{\partial x}, (t = 1 \sim 4), \quad (5)$$

where,  $k_1$  is the wave number in a vacuum,  $k_t^{(n)}$  is the propagation constants in  $x$ -direction, and they are defined by

$$k_t^{(n)} := \sqrt{k_t^2 - (-j 2 n \pi / p)^2}, \quad k_t := s \sqrt{\epsilon_{t-1} \mu_0}, \quad (6)$$

Moreover,  $E_0^{(1)}$  is incident pulse at  $x = 0$  for complex frequency domain and it can be expressed as follows:

$$E_0^{(1)} = \frac{\omega_0}{s^2 + \omega_0^2} (1 - e^{-t_w s}), \quad (\omega_0 := 2\pi / t_w). \quad (7)$$

$A_n$ ,  $B_n$  of each regions are unknown coefficients to be determined from boundary conditions. Here, to express

the dispersion media, complex dielectric constant are employed by the combination with Sellmeier formula and orientational polarization [5-7]

$$\frac{\mathcal{E}_L(s)}{\mathcal{E}_0} \triangleq \sum_{l=1}^3 \frac{\Theta_{l,L}^2}{s^2 + s\mathcal{G}_{l,L} + \omega_{l,L}^2} + \frac{\Psi_{l,L}}{1 + s\tau_{l,L}}, \quad (L=1, 2), \quad (8)$$

The parameters  $(\Theta_l, \mathcal{G}_l, \omega_l, \tau_l, \Psi_l)$  of Eq.(8) were found from Ref.[5], and here, the subscript  $L$  indicates region  $S_2$  or  $S_4$ . From the boundary condition based on Eqs.(1)-(5), we drive the simultaneous equation for reflection coefficients  $R_n$ . To obtain the  $R_n$ , we divided into a perfectly conducting region  $C_b$  and gap region  $\bar{C}_b$  ( $b=1 \sim 3$ ) at  $x=d$ , apply the boundary conditions by utilizing the point matching method(PMM) as following equation: [6,7]

$$Y_\alpha \triangleq \frac{y}{p} = \frac{\alpha}{(2N_1 + 1)}, \quad \alpha = 1 \sim (2N_1 + 1). \quad (9)$$

Therefore, boundary conditions are as follows:

$$E_z^{(1)}(0, y) = E_z^{(2)}(0, y), \quad H_y^{(1)}(0, y) = H_y^{(2)}(0, y), \quad (10)$$

$$E_z^{(2)}(d_0, y) = E_z^{(3)}(d_0, y), \quad H_y^{(2)}(d_0, y) = H_y^{(3)}(d_0, y), \quad (11)$$

$$Y_\alpha \in C_b; \quad E_z^{(3)}(d, y) = E_z^{(4)}(d, y) = 0, \quad (12)$$

$$Y_\alpha \in \bar{C}_b; \quad E_z^{(3)}(d, y) = E_z^{(4)}(d, y), \quad H_y^{(3)}(d, y) = H_y^{(4)}(d, y), \quad (13)$$

$$E_z^{(4)}(d + d_2, y) = 0, \quad (14)$$

From above Eqs.(10)-(14), we can obtain the simultaneous equation in regard to  $R_n$ .

$Y_\alpha \in C_b$  ( $b=1 \sim 3$ ):

$$\sum_{N=-N_1}^{N_1} [\xi_1^{(n)} e^{-k_3^{(n)} d_1} + \xi_2^{(n)} e^{+k_3^{(n)} d_1}] R_n e^{-j \frac{2n\pi}{p} y} = -(\xi_3^{(0)} e^{-k_3^{(0)} d_1} + \xi_4^{(0)} e^{+k_3^{(0)} d_1}) E_0^{(i)} \quad (15)$$

$Y_\alpha \in \bar{C}_b$  ( $b=1 \sim 3$ ):

$$\sum_{n=-N_1}^{N_1} [K_n^{(-)} \xi_1^{(n)} e^{-k_3^{(n)} d_1} + K_n^{(+)} \xi_2^{(n)} e^{+k_3^{(n)} d_1}] R_n e^{-j \frac{2n\pi}{p} y} = -(K_0^{(-)} \xi_3^{(0)} e^{-k_3^{(0)} d_1} + K_0^{(+)} \xi_4^{(0)} e^{+k_3^{(0)} d_1}) E_0^{(i)} \quad (16)$$

where,

$$\Gamma_{1,n}^{(+)} := 1 + k_1/k_2^{(n)}, \quad \Gamma_{1,n}^{(-)} := 1 - k_1/k_2^{(n)}, \quad \Gamma_{2,n}^{(+)} := 1 + k_1^{(n)}/k_2^{(n)},$$

$$\Gamma_{2,n}^{(-)} := 1 - k_1^{(n)}/k_2^{(n)}, \quad \Gamma_{3,n}^{(+)} := 1 + k_2^{(n)}/k_3^{(n)}, \quad \Gamma_{3,n}^{(-)} := 1 - k_2^{(n)}/k_3^{(n)},$$

$$K_n^{(-)} := k_4^{(n)} \beta_n - k_3^{(n)}, \quad K_n^{(+)} := k_4^{(n)} \beta_n + k_3^{(n)},$$

$$\beta_n := (1 + e^{-2k_4^{(n)} d_2}) / (1 - e^{-2k_4^{(n)} d_2}),$$

$$\xi_1^{(n)} := \Gamma_{2,n}^{(-)} \Gamma_{3,n}^{(+)} e^{-k_2^{(n)} d_0} + \Gamma_{2,n}^{(+)} \Gamma_{3,n}^{(-)} e^{k_2^{(n)} d_0},$$

$$\xi_2^{(n)} := \Gamma_{2,n}^{(-)} \Gamma_{3,n}^{(-)} e^{-k_2^{(n)} d_0} + \Gamma_{2,n}^{(+)} \Gamma_{3,n}^{(+)} e^{k_2^{(n)} d_0},$$

$$\xi_3^{(0)} := \Gamma_{1,0}^{(+)} \Gamma_{3,0}^{(+)} e^{-k_2^{(0)} d_0} + \Gamma_{1,0}^{(-)} \Gamma_{3,0}^{(-)} e^{k_2^{(0)} d_0},$$

$$\xi_4^{(0)} := \Gamma_{1,0}^{(+)} \Gamma_{3,0}^{(-)} e^{-k_2^{(0)} d_0} + \Gamma_{1,0}^{(-)} \Gamma_{3,0}^{(+)} e^{k_2^{(0)} d_0}.$$

The reflection coefficients  $R_n$  is found by Eqs.(15), (16), and then reflection electric fields  $E_z^{(r)}(x, y)$  obtained using the value of  $R_n$  is transformed into normalized time domain using the FILT method as following equations:[8]

$$e_z^{(r)}(T) := \frac{1}{2\pi j} \int_{\gamma-j\infty}^{\gamma+j\infty} E_z^{(r)}(S) e^{ST} dS,$$

$$= \frac{e^\alpha}{T} \left( \sum_{n=1}^{N-1} F_n - 2^{-(J+1)} \sum_{q=0}^J C_{Jq} F_{N+q} \right), \quad (17)$$

where,

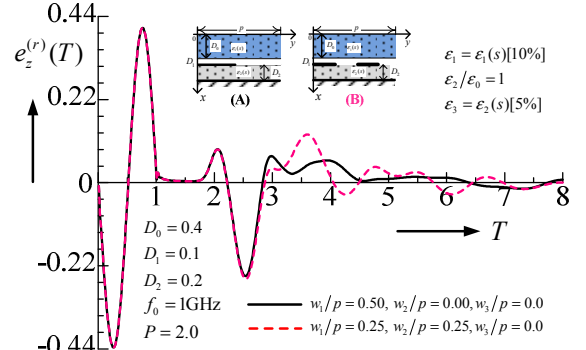
$$F_n := (-1)^n \text{Im} \left\{ E_z^{(r)} \left( \frac{a + i(n-0.5)\pi}{T} \right) \right\},$$

$$C_{JJ} := 1, \quad C_{Jq-1} := C_{Jq} + \frac{(J+1)!}{q!(J+1-q)!}.$$

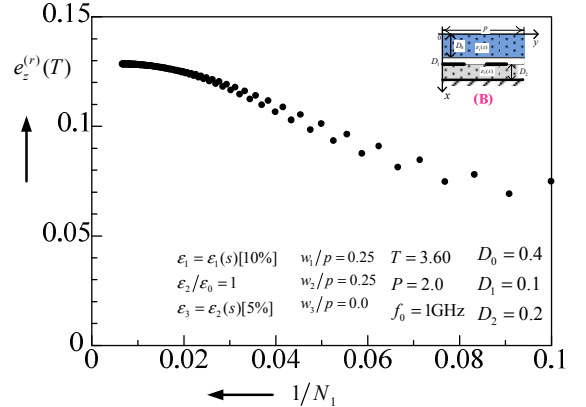
### 3. Numerical analysis

The parameter values of complex dielectric constants are used the result obtained from Ref.[5].

The values of FILT parameters chosen are  $a = 4$ ,  $N = 90$ ,



**Figure 2** Comparison of pulse responses for number of normalized conductor width



**Figure 3** Convergence of  $1/N_1$

and  $J = 10$ , and other parameters are center frequency  $f_0 = 1\text{GHz}$ , soil moisture  $\epsilon_1 = \epsilon_1(s)$  with a water ratio 10%, soil moisture  $\epsilon_3 = \epsilon_2(s)$  with a water ratio 5%, the normalized depth  $D_0 (\triangleq d_0/p) = 0.4$ ,  $D_1 (\triangleq d_1/p) = 0.1$ ,  $D_2 (\triangleq d_2/p) = 0.2$ , and truncation mode number  $N_1 = 100$ , normalized periodic length  $P (\triangleq p/(t_w c)) = 2.0$ . Figure 2 shows the comparison of pulse response for number of normalized conductor width as following two type structures:

(A)  $w_1/p = 0.5$ ,  $w_2/p = 0$ ,  $w_3/p = 0$ ,

(B)  $w_1/p = 0.25$ ,  $w_2/p = 0.25$ ,  $w_3/p = 0$ .

From Fig.2, we can see the effect of conductor number at near  $T \geq 2.8$ , and can confirm that the pulse amplitude for structure (B) becomes large at  $T \cong 3.6$  compared with

structure (A) by the influence of multiple reflections for conducting strips and different dispersion media. In order to examine the results obtained, we will investigate the analysis accuracy from convergence of truncation mode number  $1/N_1$  and fixed the FILT parameters.

Figure 3 shows the convergence of the truncation mode number  $1/N_1$  versus the normalized electric field  $e_z^{(r)}(T)$  for fixed  $T = 3.60$  as condition of Fig.2. From Fig.3, the relative error in the normalized electric fields to the extrapolated true value is less than about 1% when we computed by using the  $N_1 \geq 76$ . In this analysis, we use  $N_1 = 100$ . Therefore, we can see that it is necessary to select large mode number in this structure compared with previous structure [6,7].

#### 4. Conclusions

In this paper, we analyzed the pulse reflection response from periodically conducting strips with air region in dispersion media, and also investigated the influence of number of periodically conducting strips with reflected plate by using a combination of FILT and PMM methods.

#### 5. References

1. L. B. Felsen Ed., "Topics in Applied Physics: Transient Electromagnetic Fields," vol.10, Springer-Verlag, 1976.
2. M. Sato, "Subsurface Imaging by Ground Penetrating Radar," *IEICE Trans. Electron.*, vol.J85-C, no.7, pp. 520-530, 2002.
3. J. Sonoda, T. Kon, M. Sato, and Y. Abe, "Characteristics of Detection for Cavity under Reinforced Concrete Using Ground Penetrating Radar by FDTD Method," *IEICE Trans. Electron.*, vol.J100-C, no.8, pp. 302-309, 2017.
4. R. Persico, "Introduction to ground penetrating radar Inverse Scattering and Data Processing," Wiley, 2013.
5. R. Ozaki, N. Sugizaki, and T. Yamasaki, "Numerical Analysis of Pulse Responses in the Dispersion Media," *IEICE Trans. Electron.*, vol.E97-C, no.1, pp. 45-49, 2014.
6. R. Ozaki and T. Yamasaki, "Analysis of Pulse Reflection Responses from Periodic Perfect Conductor in Two Dispersion Media," *IEICE Trans. Electron.*, vol.E100-C, no.1, pp. 80-83, 2017.
7. R. Ozaki and T. Yamasaki, "Pulse Reflection Responses from Two Dispersion Media with Conducting Strips," *Proc. 32nd URSI-GASS*, Canada, Aug.19-27, 2017.
8. T. Hosono, "Numerical Inversion of Laplace Transform and some Applications to Wave Optics," *Radio Science*, vol.16, no.6, pp.1015-1019, 1981.