Asymptotic Analysis for Transient Scattered Magnetic Field from a Conducting Cylinder Coated with a Thin Dielectric Layer

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Abstract

By applying the Fourier transform method to each transient scattered magnetic field element represented by an integral form, we derive time-domain asymptotic solutions (TD-ASs) when a high-frequency pulse wave is incident on a two-dimensional conducting cylinder coated with a dielectric layer (2-D coated conducting cylinder). We assume that the radius of a coated conducting cylinder is sufficiently large as compared with the wavelength of a central angular frequency of a pulse source function and that a coating layer is thinner than the wavelength.

1. Introduction

The studies on the frequency-domain (FD) and time-domain (TD) scattered fields from two-dimensional (2-D) circular cylinders [1-4] with a size sufficiently larger than the wavelength have been important research subjects for a variety of applications in the area such as the radar cross section (RCS), high-resolution radar, and target identification.

In the present study, by applying the Fourier transform method [5] to a transient scattered magnetic field integral, we develop novel high-frequency (HF) TD asymptotic solutions (TD-ASs) for transient scattered magnetic field elements from a 2-D conducting cylinder coated with a dielectric layer (2-D coated conducting cylinder). Transient scattered magnetic field elements include the direct geometric optical ray (DGO), the reflected GO (RGO), the pseudo surface diffracted ray (pseudo SD), and the SD.

We assume that a coating layer is thinner than the wavelength of a central angular frequency of a pulse source function [6]. The time convention $\exp(-i\omega t)$ is adopted and suppressed in this paper.

2. Time-Domain Asymptotic Analysis

2.1 Formulation

Fig. 1 shows a 2-D coated conducting cylinder with radius $\rho = a$ covered with a medium 2 (ε_2^*, μ_0) of thickness $t\,(=a-b)$, coordinate systems (x, y, z) and (ρ, ϕ), and

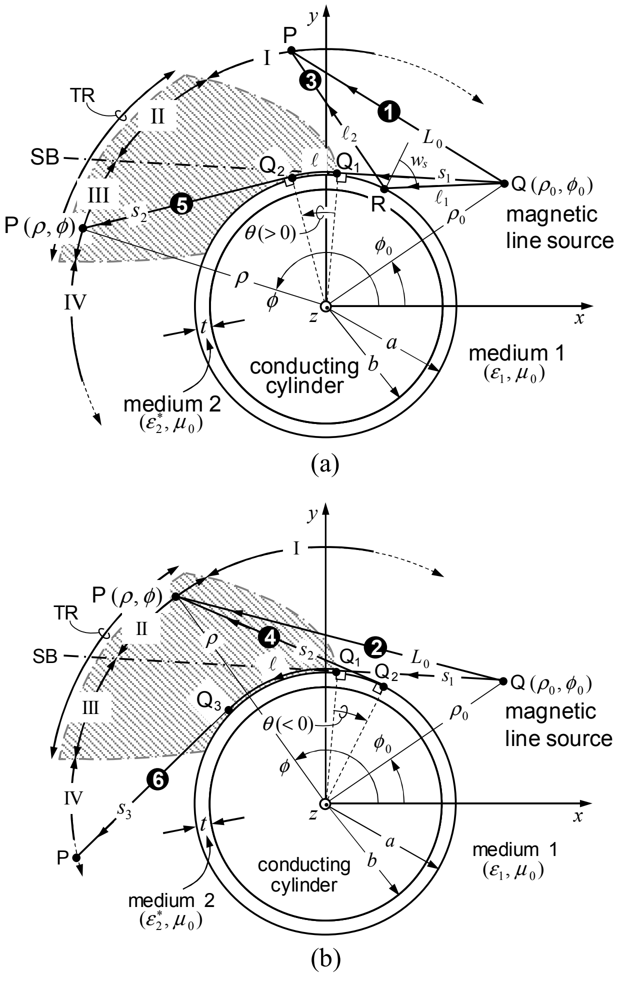

Figure 1. Scattered phenomena by a coated conducting cylinder and coordinated systems (x, y, z) and (ρ, ϕ). $\mathrm{Q}(\rho_0, \phi_0)$: magnetic line source, $\mathrm{P}(\rho, \phi)$: observation point, SB: shadow boundary, and TR: transition region. Scattered magnetic field elements: DGO (❶ and ❷), RGO (❸), pseudo SD (❹), and SD (❺ and ❻).

magnetic line source $\mathrm{Q}(\rho_0, \phi_0)$. Notation ε_2^* denotes a complex dielectric constant of the medium 2.

The surrounding space in a medium 1 (ε_1, μ_0) is divided into 4 regions by a shadow boundary (SB) and a transition region (TR) when a HF pulse wave is incident on the coated conducting cylinder from a counterclockwise direction.

Region I denotes a deep lit region far away from the SB and region II is a lit side of the TR. Regions III and IV represent a shadow side of the TR and a deep shadow region far away from the SB, respectively.

2.2 TD-ASs for Transient Scattered Magnetic Field Elements

We assume the following truncated pulse source function $s(t)$ defined by the product of the modulated wave $s_0(t)$ and the carrier wave $\exp[-i\omega_0(t-t_0)]$ whose central angular frequency is ω_0 [6]:

$$s(t)=\begin{cases} s_0(t)\exp[-i\omega_0(t-t_0)] & \text{for } 0\le t\le 2t_0 \\ 0 & \text{for } t\le 0,\ t>2t_0 \end{cases} \tag{1}$$

where t_0 denotes a constant parameter. Denoting the Fourier transform of $s_0(t)$ by $S_0(\omega)$, the Fourier transform $S(\omega)$ of $s(t)$ can be represented by

$$S(\omega)=S_0(\omega-\omega_0)\exp(i\omega_0 t_0). \tag{2}$$

An integral $\bar{y}_{z,j,n}(t),\ j=\text{DGO, RGO, pseudoSD, SD}$, for a transient scattered magnetic field element from a coated conducting cylinder is given by the inverse Fourier transform [7] of the product of a FD asymptotic solution element $\overline{H}_{z,j,n}(t)$ [8] and the Frequency spectrum $S(\omega)$ in (2).

$$\bar{y}_{z,j,n}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{H}_{z,j,n}(t)S(\omega)\exp(-i\omega t)d\omega \tag{3}$$

$$\overline{H}_{z,j,n}(\omega)=A_{z,j,n}(\omega)\exp\left[\frac{\ell_{\text{CW}}}{c_n(\omega)}+\frac{L_{z,j}-\ell_{\text{CW}}}{c_1}\right] \tag{4}$$

$$c_n(\omega)=\frac{c_1}{1+\dfrac{\sigma_{H,n}}{2^{4/3}a^{2/3}}\left(\dfrac{c_1}{\omega_0}\right)^{2/3}}\quad \left(<c_1=\frac{1}{\sqrt{\varepsilon_1\mu_0}}\right) \tag{5}$$

where $A_{z,j,n}(\omega)$ denotes the amplitude of the FD scattered magnetic field element which varies slowly as the function of the angular frequency ω, and ℓ_{CW} and $L_{z,j}$ are the propagation distance of a creeping wave (CW) and a total propagation distance, respectively. Notations $c_n(\omega)$ and c_1 in (5) denote respectively the phase velocity of nth-order CW and the one of the medium 1. Notation n is a mode order of CW and is defined by 0 when a CW doesn't exist on a total propagation path. The eigenvalue $\sigma_{H,n}$ in (5) is determined from the following characteristic equation using the Airy function $Ai(\bullet)$ [9] and its derivative $Ai'(\bullet)$:

$$Ai'(-\sigma_{H,n})-\exp(-i\pi/6)MZ_{\nu_n}Ai(-\sigma_{H,n})=0 \tag{6}$$

where notation Z_{ν_n} denotes a normalized surface impedance (see (4) in [8]) and is given approximately by

$$Z_{\nu_n}\sim i\,Z_S\frac{\sqrt{(k_2^*a)^2-\nu_n^{\ 2}}}{k_2^*a}\tan(B_{1\nu_n}-B_{2\nu_n}) \tag{7}$$

$$B_{1\nu_n}=\sqrt{(k_2^*b)^2-\nu_n^{\ 2}}-\nu_n\cos^{-1}\left(\frac{\nu_n}{k_2^*b}\right) \tag{8}$$

$$B_{2\nu_n}=\sqrt{(k_2^*a)^2-\nu_n^{\ 2}}-\nu_n\cos^{-1}\left(\frac{\nu_n}{k_2^*a}\right) \tag{9}$$

$$Z_S=\frac{Z_2}{Z_1},\quad Z_1=\sqrt{\frac{\mu_0}{\varepsilon_1}},\quad Z_2=\sqrt{\frac{\mu_0}{\varepsilon_2^*}} \tag{10}$$

$$\nu_n=k_1a+M\tau_n,\quad \tau_n=\sigma_{H,n}\exp(i\pi/3),\quad M=(k_1a/2)^{1/3}. \tag{11}$$

When a CW doesn't exist on the total propagation path, ℓ_{CW} and $c_n(\omega)$ in (4) are reduced to simple values defined by

$$\ell_{\text{CW}}=0,\quad c_0(\omega)=c_1\quad \text{for } n=0. \tag{12}$$

After substituting (2), (4), and (12) into (3) and then applying the Fourier transform method, one obtains the following TD asymptotic solutions for transient scattered magnetic field elements.

$$\bar{y}_{z,j,0}(t)\sim A_{z,j,0}(\omega_0)s(t-L_{z,j}/c_1)\quad \text{for } j=\text{DGO, RGO, pseudo SD} \tag{13}$$

$$\bar{y}_{z,\text{SD},n}(t)\sim A_{z,\text{SD},n}(\omega_0)s_0\left(t-\frac{\ell_{\text{CW}}}{v_{g,n}(\omega_0)}-\frac{L_{z,\text{SD}}-\ell_{\text{CW}}}{c_1}\right)\bullet\exp\left(-i\omega_0\left[t-t_0-\frac{\ell_{\text{CW}}}{c_n(\omega_0)}-\frac{L_{z,\text{SD}}-\ell_{\text{CW}}}{c_1}\right]\right) \tag{14}$$

$$v_{g,n}(\omega_0)=\frac{c_n(\omega_0)}{1-\omega_0\dfrac{c_n'(\omega_0)}{c_n(\omega_0)}},\quad c_n'(\omega_0)=\frac{d}{d\omega}c_n(\omega)\bigg|_{\omega=\omega_0} \tag{15}$$

It is observed from (13) that the $\bar{y}_{z,j,0}(t),\ j=\text{DGO, RGO, pseudo SD}$, propagates along the total propagation distance $L_{z,j}$ with the phase velocity c_1 coincident with the speed of light in the medium 1. While, it is clear from (14) that the CW in the SD propagates along the propagation distance ℓ_{CW} with the group velocity $v_{g,n}(\omega_0)\,(<c_1)$ defined in (15) and the phase velocity $c_n(\omega_0)\,(<c_1)$.

3. Conclusion

We have briefly derived the time-domain (TD) asymptotic solutions for transient scattered magnetic field elements when a HF pulse wave is incident on a conducting cylinder coated with a thin dielectric layer. We showed that the TD asymptotic solutions are easy to understand the physical insight of the transient scattered magnetic field elements.

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5. References

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