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On General Boundary Conditions

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Boundary conditions are essential, along with the Maxwell equations, in defining electromagnetic problems. On a boundary surface, two scalar conditions relating the field vectors are required to be satisfied.

One can classify various linear and local boundary conditions in terms of the number of free parameters involved. Let us assume here that the medium above the boundary is isotropic with parameters ε_o and μ_o, while $\mathbf{n}$ denotes the unit vector normal to the boundary surface.

As the two simplest examples, there are no boundary parameters for the PEC and PMC boundaries defined by the respective conditions $\mathbf{n}\times\mathbf{E}=0$ and $\mathbf{n}\times\mathbf{H}=0$. Also the DB boundary [1] requires no parameters for the definition, $\mathbf{n}\cdot\mathbf{D}=0$, $\mathbf{n}\cdot\mathbf{B}=0$. For an isotropic medium these equal the conditions $\mathbf{n}\cdot\mathbf{E}=0$ and $\mathbf{n}\cdot\mathbf{H}=0$.

The PEMC (perfect electromagnetic conductor) [2] boundary serves as an example of a one-parameter boundary: $\mathbf{n}\times(\mathbf{H}+M\mathbf{E})=0$, while the SH (soft-and-hard) conditions [3] $\mathbf{a}_t\cdot\mathbf{E}=0$ and $\mathbf{a}_t\cdot\mathbf{H}=0$ involve two scalar parameters of the vector $\mathbf{a}_t$ tangential to the boundary surface.

The most general case, the GBC (general boundary conditions) boundary [4] can be defined by

$$\begin{pmatrix} \mathbf{a}_1 & \mathbf{b}_1 \\ \mathbf{a}_2 & \mathbf{b}_2 \end{pmatrix}\cdot\begin{pmatrix} \mathbf{E} \\ \eta_o\mathbf{H} \end{pmatrix}=0, \tag{1}$$

where the four dimensionless vectors $\mathbf{a}_1-\mathbf{b}_2$ have no a priori restrictions and $\eta_o=\sqrt{\mu_o/\varepsilon_o}$. For a given boundary, the representation (1) is not unique since the four vectors can be changed by a linear transformation. A more unique condition can be expressed as

$$\begin{pmatrix} \alpha\mathbf{n}\cdot\mathbf{E} \\ \beta\mathbf{n}\cdot\eta_o\mathbf{H} \end{pmatrix}+\begin{pmatrix} \mathbf{a}_{1t} & \mathbf{b}_{1t} \\ \mathbf{a}_{2t} & \mathbf{b}_{2t} \end{pmatrix}\cdot\begin{pmatrix} \mathbf{E}_t \\ \eta_o\mathbf{H}_t \end{pmatrix}=0, \tag{2}$$

where $()_t$ denotes a vector tangential to the boundary. Because the dimensionless scalars α and β can be chosen, the number of parameters defining a GBC boundary is at most 8.

One can show that any given plane wave can be uniquely split in two plane waves, labeled by TE_1 and TE_2, which do not interact in reflection from the GBC boundary (1). Special cases of TE_1 and TE_2 waves satisfying the conditions (1) identically (with no reflection) are called matched waves. The same properties remain valid when the medium is more generally non-birefracting so that there is at most one reflected wave for each incident wave.

In the present paper we discuss various properties of the GBC boundary on plane waves. Also, a possibility to realize the GBC surface in terms of an interface of a certain bianisotropic medium is suggested.

References

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[4] I. V. Lindell and A. Sihvola, "Electromagnetic wave reflection from boundaries defined by general linear and local conditions," *IEEE Trans. Antennas Propag.*, **65**, 9, pp. 4656–4663, 2017.