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Diffraction by Two PEC Inverted Staggered Half Planes

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Abstract

This paper presents the formulation and the procedure to get the solution of the scattering problem constituted by two Perfectly Electrically Conducting (PEC) inverted half planes who are staggered with respect to each other.
While the Wiener Hopf equations of this problem are well known, we obtain a novel solution of them by resorting to the Fredholm factorization.

1. Introduction

In this paper we consider the classical canonical scattering problem constituted of two inverted staggered Perfectly Electrically Conducting (PEC) half planes.
Cartesian coordinates as well as polar coordinates will be used to describe the problem. Two origins are considered, see Fig. 1: the edge of the upper half-plane is chosen as origin O for coordinates (x,y,z) and (ρ, φ,z), while the edge of the lower half-plane is chosen as origin O' for coordinates (x_2,y_2,z) and (ρ_2, φ_2,z). The two reference systems are related through the following relations x_2=x-s, y_2=y+d.
Fig.1 shows the two half planes which occupy the regions x<0,y=0,-∞<z<∞ and x>s,y=-d,-∞<z<∞; the problem is with translational symmetry along the z axis. In the following we will consider s as the staggered parameters along x and d as the distance along y between the two half-planes. We consider that s can assume positive and negative values while d only positive; in Fig. 1 s is positive. While s is negative we have an overlap region where a parallel PEC plane waveguide is generated.
We define three regions: region 1 is the half space region delimited by y>0, region 2 is the rectangular region with -d<y<0 and region 3 is the half space region delimited by y<-d. For simplicity we consider the structure immersed in a homogenous dielectric medium, for example the free space.
A plane wave with azimuthal direction $\varphi=\varphi_o$ ($0\leq\varphi_o\leq\pi$) is incident on the structure in region 1. For the sake of simplicity we consider the incident field constituted of the E-polarized plane wave with longitudinal component

$$E_z^i = E_o e^{jk\rho\cos(\varphi-\varphi_o)} \qquad (1).$$

where k is the free space propagation constant.

The literature shows several works that studies this problem. In particular [1]–[2] formulate the problem as a matrix Wiener-Hopf equation and it is solved by using the weak factorization method. These formulations highlight the presence of problematic exponential behavior of the spectral unknowns depending on the staggered parameter s. Other similar problems are described in [3-7].
In this work we suggest an alternative approach, the Fredholm factorization [8,9], to handle this problem.
The spectral formulation of the problem reported in the following sections will take into consideration the interaction at near field of the edges at (x,y)=(0,0) and (x,y)=(s,-d) by using a correct and comprehensive model. The formulation is derived via the Wiener-Hopf (WH) technique [9], the Fredholm factorization [8] and the recently developed modelling through equivalent circuits[10-11].

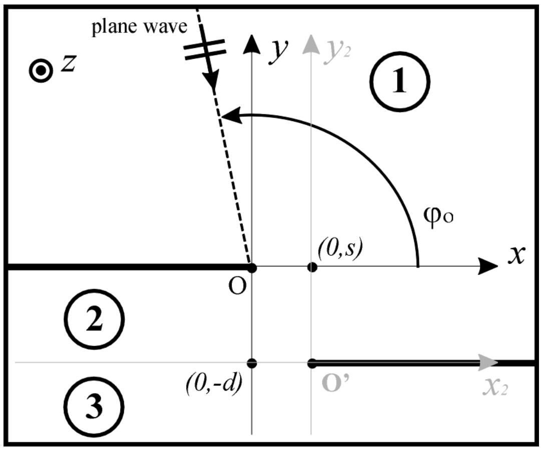

Figure 1. The two PEC Inverted Staggered Half Planes illuminated by a plane wave.

The solution of the problem is obtained through the following steps:
1. formulation via coupled Wiener-Hopf equations (WHEs),
2. reduction to integral equations using Fredholm factorization with circuital representations,
3. numerical estimation of the spectra and analytical properties of the unknowns,
4. asymptotic estimations of electromagnetic field.

2. Wiener-Hopf Formulation and Hints for the Solution

For the homogenous region 1 with PEC face at x<0, y=0 the following WH equation (2) holds [9,12-13] in terms WH unknowns (3) defined at y=0.

$$I_{a-}(\eta)+I_{1+}(\eta)=Y_{\infty}(\eta)V_{1+}(\eta) \qquad (2).$$

$$\begin{aligned}
V_{1+}(\eta)&=\int_0^{\infty}E_z(x,0)e^{j\eta x}dx,\\
I_{1+}(\eta)&=\int_0^{\infty}H_x(x,0)e^{j\eta x}dx\\
I_{a-}(\eta)&=\int_{-\infty}^{0}H_x(x,0_+)e^{j\eta x}dx\\
V_{1\pi+}(\eta)&=\int_{-\infty}^{0}E_z(x,0_\pm)e^{-j\eta x}dx=0\\
I_{1\pi+}(\eta)&=-\int_{-\infty}^{0}H_x(x,0_-)e^{-j\eta x}dx
\end{aligned} \qquad (3).$$

For the homogenous region 2 with PEC face at x<0, y=0 and x>s, y=-d the following spectral equation (4) holds [9,14] in terms Fourier transforms (5) according to the transmission line modelling of plane stratified media.

$$\begin{aligned}
i(\eta,0_-)&=-Y_{11}(\eta)v(\eta,0)-Y_{12}(\eta)v(\eta,-d) \qquad (4).\\
i(\eta,-d_+)&=Y_{21}(\eta)v(\eta,0)+Y_{22}(\eta)v(\eta,-d\ )
\end{aligned}$$

$$\begin{aligned}
v\ (\eta,y)&=\int_{-\infty}^{\infty}E_z(x,y)e^{j\eta x}dx \qquad (5).\\
i\ (\eta,y)&=\int_{-\infty}^{\infty}H_x(x,y)e^{j\eta x}dx
\end{aligned}$$

Note that $Y_{11}(\eta)=Y_{22}(\eta)=-jY_{\infty}(\eta)\cot[\xi(\eta)d]=Y_l(\eta)$ and $Y_{12}(\eta)=Y_{21}(\eta)=Y_m(\eta)=j\dfrac{Y_{\infty}(\eta)}{\sin[\xi(\eta)d]}$, $Y_{\infty}(\eta)=\dfrac{\sqrt{k^2-\eta^2}}{k\,Z_o}$, $\xi(\eta)=\sqrt{k^2-\eta^2}$.

We rewrite (4) as (7) by defining the following WH unknowns at y=-d

$$\begin{aligned}
V_{2+}(\eta)&=\int_0^{\infty}\dot{E}_z(x_2,0)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^{\infty}E_z(x,-d)e^{j\eta x}dx=0\\
I_{2+}(\eta)&=\int_0^{\infty}\dot{H}_x(x_2,0_+)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^{\infty}H_x(x,-d_+)e^{j\eta x}dx\\
I_{d+}(\eta)&=\int_0^{\infty}\dot{H}_x(x_2,0_-)e^{j\eta x_2}dx_2=e^{-j\eta s}\int_s^{\infty}H_x(x,-d_-)e^{j\eta x}dx\\
V_{2\pi+}(\eta)&=\int_{-\infty}^{0}\dot{E}_z(x_2,0)e^{-j\eta x_2}dx_2=e^{j\eta s}\int_{-\infty}^{s}E_z(x,-d)e^{-j\eta x}dx,\\
I_{2\pi+}(\eta)&=-\int_{-\infty}^{0}\dot{H}_x(x_2,0_+)e^{-j\eta x_2}dx_2=-e^{j\eta s}\int_{-\infty}^{s}H_x(x,-d_+)e^{-j\eta x}dx
\end{aligned} \qquad (6).$$

$$\begin{aligned}
-I_{1+}(\eta)+I_{1\pi+}(-\eta)&=Y_{11}(\eta)V_{1+}(\eta)+Y_{12}(\eta)\exp(j\eta s)V_{2\pi+}(-\eta)\\
I_{2+}(-\eta)-I_{2\pi+}(\eta)&=Y_{21}(\eta)\exp(j\eta s)V_{1+}(-\eta)+Y_{22}(\eta)V_{2\pi+}(\eta)
\end{aligned} \qquad (7).$$

Finally region 3 is modelled similarly to region 1 with the WH equation

$$-I_{d+}(-\eta)+I_{2\pi+}(\eta)=Y_{\infty}(\eta)V_{2\pi+}(\eta) \qquad (8).$$

Using the Fredholm factorization [8], the coupled WH equations (2), (4), (8) provides, by eliminating the current unknowns, Fredholm integral equations whose unknowns are the two non-trivial voltage $V_{1+}(\eta),V_{2\pi+}(\eta)$. Simple numerical discretization of the system of Fredholm integral equations, as sample and hold, enhanced by contour warping is performed for the evaluation of the voltage unknowns.

The integral representations obtained during the application of the Fredholm factorization procedure also provide the currents through the knowledge of the voltages.

The analytical properties of the approximate voltages, directly obtained from the solution of the system of Fredholm integral equations, allow the asymptotic estimation via UTD [15] of the total field in region 1 an 3.

In particular we note that the voltages $V_{1+}(\eta),V_{2\pi+}(\eta)$ are spectral transformation of the E_z field along x at y=0 and y=-d.

Numerical validations and results will be shown during the presentation.

3. Acknowledgements

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