



A Combined Ensemble Kalman and Particle Filter for Ionospheric Data Assimilation

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The ionosphere is a complex and rapidly changing environment with complex statistical properties. Nonetheless there are a wide range of ionospheric data assimilation models in development and use across the globe. The fundamental idea of a data assimilation scheme is to make a prediction of the current state of the ionosphere and then update this prediction with data. The main limitation with existing ionospheric DA schemes is that the most common techniques assume that the model errors have a Gaussian probability distribution function (pdf). This serves as a fundamental limit to the dynamics that the ionospheric data assimilation models can represent.

In reality, the probability distributions of the underlying ionospheric system are not Gaussian. One way to fully deal with the system would be to use a particle filter (PF). A PF is a Monte Carlo method which approximates the true probability distribution function. The reason that PFs have not been implemented for the ionospheric (and other geophysical) systems is due to the very large computational expense. The number of particles required to accurately specify the underlying pdf scales exponentially with the number of variables in the state vector. Furthermore, it has been estimated that the number of particles required for use in a geophysical model is between 1% and 10% of the state vector [van Leeuwen, 2009]. Since, a “standard” model resolution of 5 degrees in longitude and latitude and ~100 altitude levels which would mean 2,500 to 25,000 particles would be required. If the particles were realisations of a physics based model only a handful of the world’s supercomputers would be able to handle the computational cost in real time.

MMaDAM (Malvern Maths Data Assimilation Model) is a new generalised data assimilation system which is a hybrid approach combining the commonly applied ensemble Kalman filter techniques and the, as yet unused in geophysics, particle filter techniques. The prediction step in the DA system involves allowing the particles (samples / ensemble members) to evolve according to the dynamics of the system (e.g. by using a physics based model to propagate densities forward). It is the computational cost of this step which limits how many particles can be used. The update step involves modifying the prediction based on the data. It is the implementation of this step where different approaches arise, and the EnKF and PF are two such approaches. MMaDAM combines the EnKF and PF by using the “progressive correction” approach described by Musso et al. [Musso et al., 2001] and Frei and Künsch [Frei and Künsch, 2013]. This approach uses a damped ensemble Kalman filter to partially update the prediction and then a damped particle filter is also used to further update the system. The first part (EnKF) of the update relies on the standard Gaussian assumptions; however, the second part (PF part) does not. Thus MMaDAM can capture some of the non-Gaussian features of the probability distributions.

This paper describes how MMaDAM has been modified to use NeQuick [Nava et al., 2008] as its background model so that it can be applied to ionospheric data assimilation and forecasting. The new ionospheric model (TRAIN) uses an ensemble of NeQuick models as particles.

References

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