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Mutual Coupling Modeling and Calibration in Antenna Arrays for AOA Estimation

VO T. T.(1), OUVRY L. (1), SIBILLE A. (2) and BORIES S. (1)
(1) Univ. Grenoble Alpes, CEA, LETI, MINATEC Campus, F-38000, Grenoble, France
(2) LTCI, Telecom ParisTech, University of Paris-Saclay, Paris, France

Abstract

In antenna arrays, mutual coupling is an undesired effect which degrades the array pattern and the performance of array signal-processing algorithms like for Angle-Of-Arrival (AOA) estimation. Various approaches for mutual-coupling compensation that evaluate the coupling in receiving and transmitting modes have been studied. In this paper, a definition of the mutual coupling in receiving mode from the scattering parameters is introduced. A dissimilarity factor is added on the signals after the coupling calibration to simulate amplitude and phase variations at the termination of array elements. The coupling model shows the role of antenna characteristics and distance between the elements. When the antennas are all connected to a matched load and there is no dissimilarity between the antennas, the coupling in the receiving mode assimilates to that in the transmitting mode. To validate this statement, the coupling of an array of two patch antenna elements in these two modes is calculated, which shows an adequate agreement. A simplified technique calibrating the coupling and the dissimilarity is also proposed. The application of these coupling-calibration techniques together shows many possibilities to improve antenna arrays processing.

1 Introduction

The Angle-Of-Arrival (AOA) of a signal source or of an obstacle echo in a RADAR context is an important information, which can be estimated using an antenna array. However, AOA-finding algorithms such as MUSIC [1] and time/phase based direction finding [2] require accurate knowledge of the received signal. In practical arrays, this requirement is difficult to meet because of the mutual coupling between the antenna elements and the dissimilarity of the signal amplitude and phase between antennas, which degrade the AOA-finding performance. Hence it is important to study compensation methods to suppress such array imperfections.

Calibration methods taking into account mutual coupling were introduced in [3]. In these methods, the coupling model is analyzed when the array elements are either in receiving or in transmitting modes. In the receiving mode, all radiator elements are ideally matched and receive incident wave from a transmitting antenna in the far field [4, 5, 6]. However it is not always practical to make measurement with such a far field antenna and is definitely out-of-scope at production steps. In [7], the authors use the measurements obtained with a transmitting antenna at twelve known angles of arrival from 0^o to 180^o to calibrate not only the coupling but also the elements dissimilarities. This method is useful in a high resolution direction finding system.

In [8], the mutual coupling is calculated in the transmitting mode as one antenna in the array plays the role of the transmitting antenna and the others the role of the receiving antennas. This approach does not require any external antenna, but it analyzes the array in open-circuit conditions, which usually has matched loads in practice. It is also shown that the coupling path in the receiving and transmitting modes are different in length. However we will show that this difference can be compensated if the length difference between these two paths is known.

In this article we analyze the antenna array in the transmitting mode, aiming to simplify the measurement procedure and to apply matched load impedances on the array, similarly to the receiving mode. We then compare the coupling factor calculated with our method to that we found from a method in the receiving mode, based on measurement data at a known angle of arrival with an external transmitting antenna, as in [4]. We propose also an extended coupling model, which can calibrate the coupling and the dissimilarity from only two measurements at distinct angles of arrival.

2 Mutual Coupling in Receiving Mode

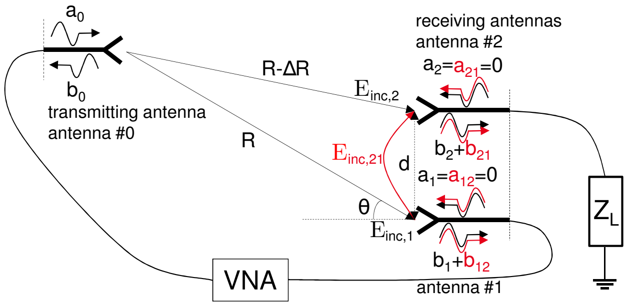

Figure 1. Coupling mechanism.

Consider for simplicity a two elements antenna array, where all elements are terminated with a matched load. The incident wave and the received wave on each element are shown in Figure 1. Since the receiving wave can be converted into a voltage at the antenna terminal, the relation between the measured wave (measured signal) and the incoming wave (incoming signal) alone can be expressed through the receiving-mutual-coupling-impedance [3]:

$$\begin{bmatrix} b'_1 \\ b'_2 \end{bmatrix}(\theta) = \begin{bmatrix} 1 & z \\ z & 1 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}(\theta) \tag{1}$$

where the measured wave b'_{i} is the sum of the incoming signal due to the incident field $\mathbf{E}_{\mathrm{i}}$, b_i, and the receiving wave caused by the coupling, b_{ij} (see Figure 1).

The signals b_1 and b_2, can be calculated from the incoming fields, $\mathrm{E}_{inc,1}$ and $\mathrm{E}_{inc,2}$, which are shifted in phase by $\Delta\phi = k\Delta R = kd\sin\theta$, and from the antenna characteristics, which may not all be identical. (k is the wave number). The detailed calculation can be found in [9]. Here we propose a dissimilarity model between the antenna terminations:

$$b_2(\theta) = m_{21}(\theta)\, b_1(\theta) \exp(jkd\sin\theta) \tag{2}$$

where $\mathrm{m}_{21}(\theta)$ is a complex number representing the amplitude and the phase dissimilarity between receiving antenna terminals. Equations 1 and 2 form the extended coupling model.

The calibration from the far field measurements at twelve angles of arrival suppress significantly the coupling and the dissimilarity [7]. However, because of its complexity, we do not use this method in this paper. We will show later that the coupling factor z, can be calculated from the transmission coefficient between the array elements, in the absence of antenna dissimilarity. To make a fair comparison between our method and existing ones, we will compare the former to the method calibrating only the coupling by assuming that the antennas are identical ($m_{21}(\theta) = 1$) [4, 5, 6].

In practice, the coupling is not the only problem in the antenna array but also the amplitude and phase dissimilarity. We then take a further step to calibrate the variation regardless of θ, by assuming that $m_{21}(\theta)$ is a constant, so that the coupling and the dissimilarity can be determined after two measurements at known (distinct) angles of arrival, by solving equations 1 and 2.

3 Mutual Coupling in Transmitting Mode

The coupling factor, z, is defined as the ratio of the mutual coupling signal to the incoming signal alone at one antenna.

$$z = \frac{b_{21}}{b_1} = \frac{b_{12}}{b_2} \tag{3}$$

It can be calculated if the antenna characteristics are known. Although the coupling is a near field effect, we made use of the far field approximation, considering the resulting error on the AOA to be a second order effect. Thus:

$$b_{21}(\theta) = b_1(\theta) H_{\mathrm{RX},1}(\frac{\pi}{2}) H_{\mathrm{RX},2}(\frac{-\pi}{2}) \frac{-j\lambda}{4\pi d} \exp(-jkd) \tag{4}$$

where $\mathrm{H}_{\mathrm{RX},1}(\theta)$, $\mathrm{H}_{\mathrm{RX},2}(\theta)$ are the antenna transfer functions described in [9] and $\theta = \pi/2$, $\theta = -\pi/2$ for antennas #1 and #2, respectively, for a planar array. From equations 3 and 4:

$$z = H_{\mathrm{RX},1}(\frac{\pi}{2}) H_{\mathrm{RX},2}(\frac{-\pi}{2}) \frac{-j\lambda}{4\pi d} \exp(-jkd) \tag{5}$$

This equation allows to calculate the coupling factor from the antenna characteristics. However, the method always requires either measurements with an external antenna or far field simulation data to obtain these characteristics.

Since in many antennas, a phase center exists even approximately [10], for the identical receiving antennas, at the common reference plane of phase centers, we consider the following simplification:

$$phase(H_{\mathrm{RX},1}) = phase(H_{\mathrm{RX},2}) \approx 0^o \; \forall \theta \tag{6}$$

From equation 5 and 6, the phase of the coupling factor depends only on the distance between array elements.

As long as the antennas are identical, the secondary excitation of antenna #1 towards antenna #2 in the receiving mode (Figure 1) and in the transmitting mode (Figure 2) are the same. We can then simulate this process by injecting a signal $a_{1,S}$ in antenna #1 and by measuring the incoming wave caused by mutual coupling in antenna #2, as shown in the Figure 2. The coupling factor can be written as:

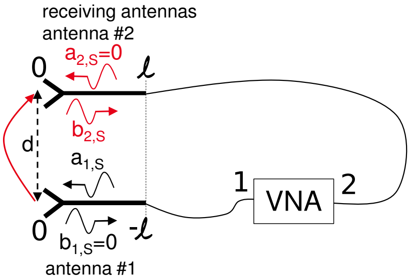

Figure 2. Transmission coefficient measurement between array elements.

$$\begin{aligned} z &= \frac{b_{2,S}(0)}{a_{1,S}(0)} \\ &= \frac{b_{2,S}(\ell)}{a_{1,S}(-\ell)} \exp(+2jk\ell) = S_{21} \exp(+2jk\ell) \end{aligned} \tag{7}$$

where position 0 refers to the antenna phase center, and positions ℓ and $-\ell$ refer to the feeder positions.

$$\begin{aligned} |z| &= |S_{21}| \\ phase(z) &= 3\pi/2 - kd \end{aligned} \tag{8}$$

From the coupling model and the measurement of S_{21} between the two antennas, we can calculate the coupling factor. One should note that the coupling model in equation 8 is valid in the situation where the antenna elements are identical and have the phase centers (on the same plane) independent from θ.

Once the coupling term is determined, the decoupling matrix is the inverse of the coupling matrix.

4 Results and Validation on AOA Estimation

To validate the analysis in the receiving and transmitting mode and evaluate the performance of the calibration on the AOA estimation, an array of patch antenna elements is fabricated. Figure 3 shows an antenna array with 3 receiving elements (RX0, RX1 and RX2) designed for AOA-finding used in high-resolution Radar applications. As an example of coupling and dissimilarity calibration, we analyze only the two elements array, RX0 and RX1, where the distance between elements is $d = \lambda_0/2$ at 8 GHz.

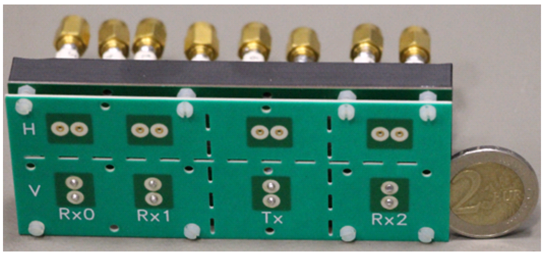

Figure 3. Photo of differential patch antennas array for AOA-finding at 7 GHz - 9 GHz

When the coupling factor and the dissimilarity factor (if it is taken into account) are determined, one can calculate the angle of arrival from equation 2 :

$$\theta = \arcsin \frac{\text{phase}(b_2) - \text{phase}(m_{21}) - \text{phase}(b_1)}{kd} \qquad (9)$$

where b_1 and b_2 are received signal at antenna RX0 and RX1, respectively. Figure 4 shows the coupling factor in an array of patch antennas calculated in the receiving mode, and in the transmitting mode with the assumption that the two antennas are identical and terminated with a matched load. In the reception based method, the coupling factor is calculated from a measurement data at a known angle of arrival, e.g. -20^o. In our transmission based method, the coupling factor is calculated from S_{21} and the distance between antenna elements, as in equation 8. The discrepancy between the two methods is explained by the fact that the two antennas are not exactly identical (edge diffraction for Rx0) and are not perfectly matched over the 7 - 9 GHz bandwidth.

Figure 5 and 6 show the estimated angle of arrival and the accuracy of the AOA algorithm (equation 9). This algorithm is applied on the measurement data at 8 GHz and from -40^o to 40^o in azimuth. Here we applied the coupling calibration method in both receiving and transmitting modes, as well as the coupling and dissimilarity calibration method in the receiving mode. Regarding the coupling calibration, the calibration in receiving mode is more efficient than the one in transmission because it takes into account the fact that the antennas may not have the phase centers (on the same plane). Regarding the coupling and dissimilarity calibration, the calibration at two known angles of arrival (for example -20^o and 20^o) highly improves the accuracy. We can see that, in this array, the dissimilarity due to edge diffraction degrades significantly the estimation. It should be noted that the calibration at two angles of arrival assumes that the dissimilarity factor m_{21} is constant regardless of the angle of arrival, which is only an approximation. Thus the discrepancy is still significant, particularly in the vertically polarized patch antennas.

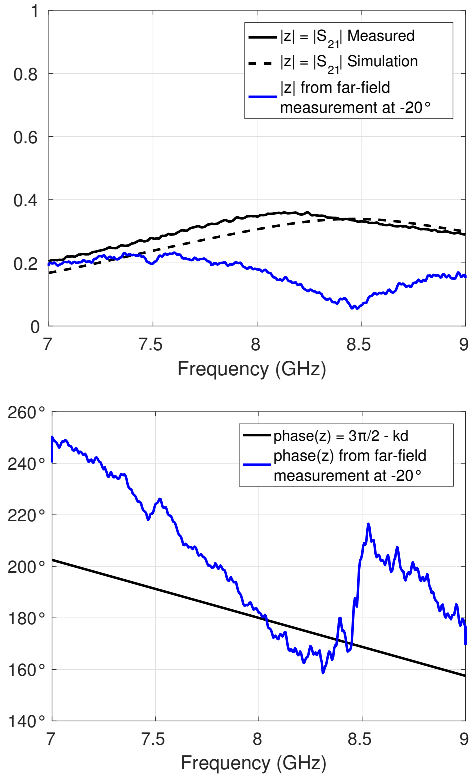

Figure 4. Coupling factor of a patch antennas array in horizontal polarization.

5 Conclusion

In this paper, the mutual coupling was first analyzed in the receiving mode. We introduced the extended coupling mode to calibrate not only the coupling but also the dissimilarity of signals at antenna terminals. The mutual coupling was then analyzed in the transmitting mode with a matched load, showing the similarity to the coupling mechanism in the receiving mode. The calculation of coupling in a transmitting mode is proposed, requiring only the transmission coefficient measurement between antennas and the knowledge of their distance. The calculation assumes the existence of a phase center for the antennas (on the same plane), and is less accurate than other classical coupling calibration methods operating in the receiving mode. The coupling and the dissimilarity due to the edge effect together were calibrated by a reception based method with two measurements at known angles of arrival. This calibration significantly

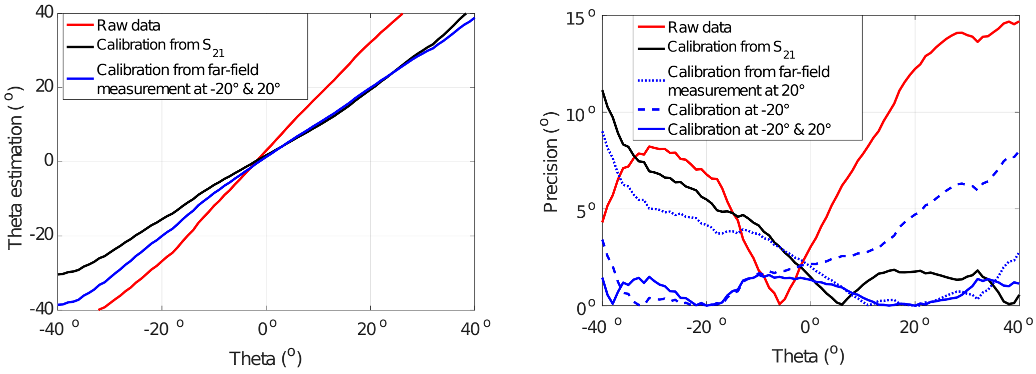

Figure 5. AOA estimation after the calibration with $[0,\lambda_0/2]$ horizontal patch antennas array.

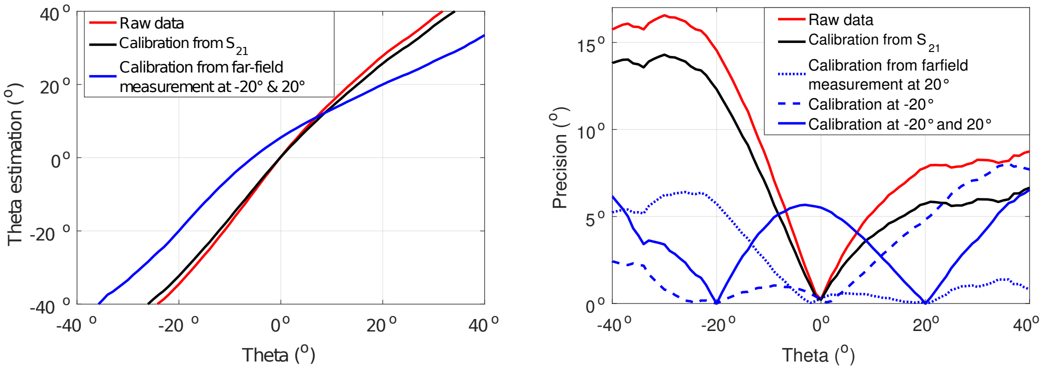

Figure 6. AOA estimation after the calibration with $[0,\lambda_0/2]$ vertical patch antennas array.

improves the AOA-finding performance, mostly when the dissimilarity does not vary with the angles of arrival. The results show the existence of many possibilities to adapt the calibration method to system requirements.

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