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Mimicking Complex Source Point Fields via Complex Transformation Optics

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Abstract

We utilize complex transformation optics (CTO) to mimic complex source point (CSP) fields. It is shown that the CSP fields can be generated via planar metamaterial slab associated with the proper complex coordinate transformations. CTO extends the real-valued coordinate transformations to complex-valued coordinate transformations. Of conventional TO, one can also control the amplitude of the fields in addition to their phase paths. In the present paper, we demonstrate that CSP fields can be produced through appropriate material tensors prescribed by the corresponding complex coordinate transformations.

1 Introduction

It is well-known that if a point source is (mathematically) located on a complex-valued point then the resulting field is well approximated by Gaussian beam in the paraxial regime [1, 2, 3]. The field due to the complex source point (CSP) is an exact solution of Maxwell's equation that can be obtained via analytic continuation to complex coordinates. Although the solution is an exact solution, the source itself is of course non-physical (hypothetical). Thus it is often used to model Gaussian beams.

Transformation optics (TO) is a powerful tool to design novel electromagnetic devices [4, 5, 6]. Relying on the form invariance property of Maxwell's equations, one can mimic a change on the metric of the space via equivalent material tensors (transformation media). This enables the design of devices with extreme control over the path of electromagnetic waves such as invisibility cloaks, perfect lens, (pseudo) black holes and other new functionalities. This control over the path of electromagnetic waves was recently extended via complex coordinate transformation (CTO) [7, 8, 9] enabling the amplitude control as well. In this case, the additional degrees of freedom and the ensuing material loss or gain provide new functionalities that are beyond the reach of conventional (real-valued) TO. In particular, it was recently shown that the complex source point (CSP) can be mimicked using parity-time ($\mathscr{P}\mathscr{T}$) metamaterials [9]. Such transformation has a mirror symmetric imaginary part that results in a balanced loss/gain, i.e. $\varepsilon(r) = \varepsilon^*(-r)$, transformation media. In this paper, we extend such concept to non-symmetric coordinate transformations and show that by using a doubly anisotropic gain-media, one can still mimic CSP fields [10]. We study the radiation characteristics of point sources in the presence of transformation media derived via CTO.

2 Complex Source Point and Complex Transformation Optics

Throughout the paper we work in the frequency domain and use $e^{-i\omega t}$ time convention. CSP is a well-known concept in EM and optics to mathematically model Gaussian beams. In CSP formalism, a point source is placed (hypothetically) on a complex-valued position. In this case, the free space Green's function can be analytically continued to complex coordinates. Note that although the source is not real (hypothetical), the resulting field is an exact solution of Maxwell's equation. Consider a line source placed at $(x_{csp}, y_{csp}) = (ib, 0)$ with b being a positive real number representing the beam parameter. The field due to the line source $\vec{J} = \hat{z}\delta(x - x_{csp})\,\delta(y - y_{csp})$ can be found as

$$\vec{E} = -\hat{z}\frac{I_0 k_0 \eta_0}{4} H_0^{(1)}\left(k_\rho \tilde{R}\right) \tag{1}$$

where $H_0^{(1)}$ is the Hankel function of first kind and zeroth order and $\tilde{R}$ is the complex distance function given as

$$\tilde{R} = \sqrt{(x - x_{csp})^2 + (y - y_{csp})^2}. \tag{2}$$

$\tilde{R}$ is a multivalued function and in order to obtain a single valued function a branch cut has to be chosen. The particular choice of $\mathbf{Re}(\tilde{R}) > 0$ is known as the source-type solution and the resulting field can be approximated (in the paraxial regime) as Gaussian beam for $x > 0$ with weak radiation for $x < 0$. In order to obtain the CSP field using CTO, one needs to mimic the complex distance function properly. For that purpose, consider the following coordinate transformation from physical space (x, y, z) to auxiliary space (x', y, z),

$$x'(x) = \begin{cases} x'_0 + d/2 + x & x \le -d/2 \\ \int_{d/2}^{x} s_x(x)dx + d/2 & -d/2 \le x \le d/2 \\ x & d/2 \le x \end{cases} \tag{3}$$

where $x'_0 = x'(-d/2)$, $s_x(x) = a_x(x) + i\sigma_x(x)$ is a complex stretching factor, and d is the slab thickness. Using the (C)TO approach, the associated constitutive tensors are obtained as $[\varepsilon] = \varepsilon_0[\Lambda]$ and $[\mu] = \mu_0[\Lambda]$, with

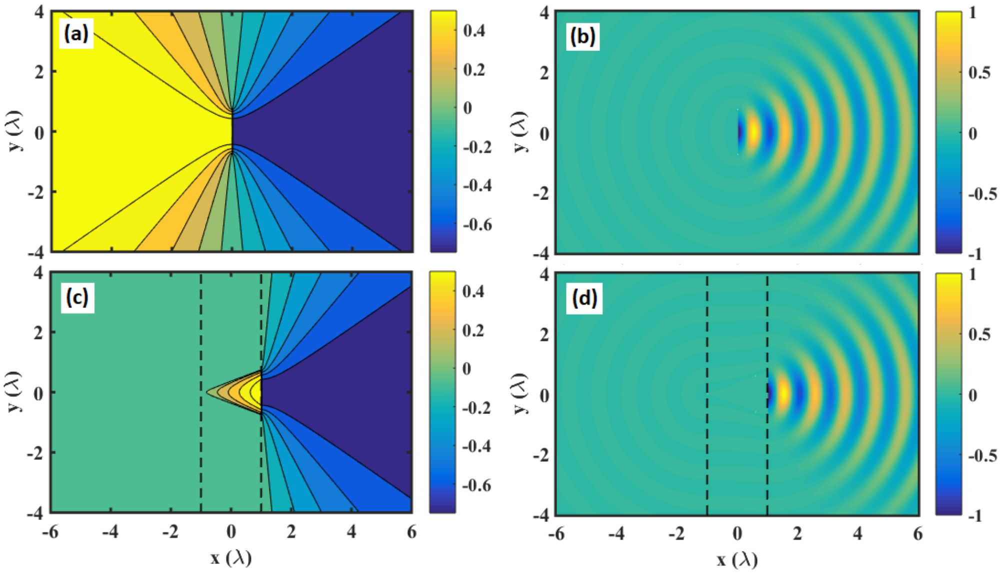

Figure 1. Realization of Gaussian beams via CSP and CTO. (a) Contour plot of $\mathrm{Im}(\tilde{R})$ for a CSP placed at $(x_{csp}, y_{csp}) = (ib, 0)$. (b) Field distribution of $\mathrm{Re}(E)$ for a CSP. (c) Contour plot of $\mathrm{Im}(R')$ based on the coordinate transformation given in Eq. 3. (d) Field distribution of $\mathrm{Re}(E)$ for a line source placed at $(x_s, y_s) = (-\lambda, 0)$ using Eq. (4,5). Dashed black lines indicates the slab region. In this example we have chosen $d = 2\lambda$, $b = 0.75\lambda$, $a_x = 0$, and $\sigma_x = b/d$.

$[\Lambda] = \det([S])^{-1}[S][S]^T$, where $[S]$ is the Jacobian of the transformation in Eq. (3) [4, 5].

The field solution $\left(\vec{E}'\right)$ in the transformed coordinates (x', y, z) can be found via analytic continuation of the known Green's function. In the transformed coordinates, the field due to a line source of $\vec{J}' = \hat{z}\delta(x' - x'_s)\,\delta(y' - y'_s)$ can be obtained analytically using Eq. 1. Different from $\tilde{R}$ defined in Eq. 2, in the transformed coordinates, the complex distance is given by

$$R' = \sqrt{(x' - x'_s)^2 + (y' - y'_s)^2} \qquad (4)$$

where (x'_s, y'_s) is the source location in the transformed coordinates associated to a source at (x_s, y_s) after the transformation given by Eq. 3. In order to obtain the proper field solution, a branch cut with $\mathbf{Re}[R'] > 0$ (the so-called source-type solution) is chosen [1, 2, 3]. In physical space, the actual fields $\vec{E}$ can be found simply [5, 4] by

$$\vec{E} = \left[S^{-1}\right]^T \cdot \vec{E}'. \qquad (5)$$

3 Results and Discussions

Fig. 1 shows the Gaussian beam generation via CSP and CTO formalism. Fig. 1(a,b) show the contour plot of $\mathbf{Im}(\tilde{R})$ and real part of electric field E_z of a CSP respectively. Note that there is a branch cut $-b < y < b$ where the $\mathbf{Im}(\tilde{R})$ changes sign when crossed this cut. Gaussian behavior is apparent from Fig. 1(b). Note also that the field is discontinuous at the branch cut. On the other hand, Fig. 1(c,d) show the contour plot of $\mathbf{Im}(R')$ and real part of electric field E_z

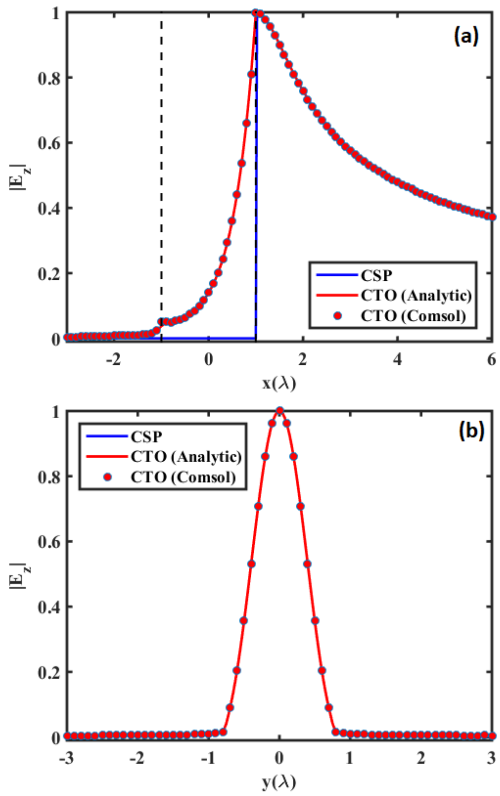

Figure 2. Comparison of CSP and CTO produced fields. (a) $|E_z|$ at $y = 0$ horizontal cut. (b) $|E_z|$ at $x = \lambda + \lambda/100$ horizontal cut. In this example we have chosen $d = 2\lambda$, $b = 0.75\lambda$, $a_x = 0.001$, and $\sigma_x = b/d$. In addition the CSP is shifted to $(x_{csp}, y_{csp}) = (\lambda + ib, 0)$.

derived via CTO respectively. Note that in this case, the physical space (x,y,z) is continuously mapped to an auxiliary space (x',y,z). Although in physical space the source is located in real-valued position (x_s,y_s), in the transformed coordinates it appears to reside at a complex-valued position (x'_s,y'_s). Because the field behavior in the transformed coordinates is defined by the distance function R', the resulting field behaves as Gaussian-like. When properly designed, the field behavior of CSP can be exactly mimicked by metamaterial slabs (see Fig. 1(c,d) and Fig.2). Note that similar to Fig. 1(a), R' has also a branch cut at $x = d/2$ between $-b < y < b$. In order to avoid this branch cut, one may choose $a_x << 0$. In the given example, we used $b = 0.75\lambda$ and $a_x = 0$ and $\sigma_x = b/d$, and $d = 2\lambda$.

Fig. 2 shows a quantitative comparison of the field behavior of CSP and metamatarial slabs. Fig. 2(a,b) shows $|E_z|$ at $y = 0$ and $x = \lambda + \lambda/100$ cuts respectively. Note that in this case, the position of CSP is shifted to $x_{csp} = \lambda + ib$ in order to align the field behavior. Also, we used $a_x = 0.001$ in order to avoid the branch cut related discontinuity. In addition the analytical results based on CTO is compared with numerical results obtained via Comsol$^{\mathrm{TM}}$. In the numerical simulations we have used metamaterial tensors $([\varepsilon],[\mu])$ with $[\Lambda] = \mathrm{diag}(id/b, -ib/d, -ib/d)$ derived by the transformation in Eq.3.

4 Conclusions

We studied the generation of Gaussian beam using CTO. We extended the real valued coordinate transformation into complex valued coordinate transformation to realize a CSP fields. It was shown that by properly designed gain-only transformation media, one can realize CSP fields. analytic

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