



## On Q-factors for thin periodic antenna and scatterer arrays

Andrei Ludvig-Osipov\* and B.L.G. Jonsson  
KTH Royal Institute of Technology, School of Electrical Engineering and Computer Science,  
100 44 Stockholm, Sweden

### 1 Extended Abstract

The Q-factor is used to estimate the operational bandwidth of electromagnetic devices, which dates back to [1]. For antennas, Yaghjian and Best [2] showed that the Q-factor  $Q_{Z'}$ , calculated from the frequency derivative of antenna's input impedance, is inversely proportional to the fractional impedance bandwidth. In [3] it was demonstrated that the Q-factor  $Q_W$ , formulated with stored energies, is close to  $Q_{Z'}$  in many cases, in particular when  $Q \geq 5$ , and thus  $Q_W$  is applicable to the antenna impedance bandwidth estimation. The stored energy approach expresses the Q-factor in terms of sources, *i.e.* electric and magnetic currents. A computationally efficient method to calculate the Q-factor of single port antennas via stored energies was introduced in [4, 5]. In [6] it is shown that the bandwidth optimization with respect to the current densities on antennas can be formulated as a convex optimization problem. A larger class of optimization problems was investigated in [7, 8] including non-convex far-field constraints.

In the present case, we investigate thin 2D periodic array antennas and scatterers. The periodicity allows a unit cell representation of the structure. We assume that structure is lossless, placed in free space and made of PEC, the equivalent magnetic sources are here neglected. We approach the stored energy via the potential representation. The Q-factor is expressed explicitly in terms of electric currents and charges. This enables us to develop tools similar to *e.g.* [6, 7, 8] to investigate bandwidth bounds for periodic structures. The bounds can be established for various constraints, such as the structure's shape, the front-to-back ratio, the equivalent far-field pattern properties, or the input impedance. These bounds are applicable to, for example, FSS-based filters, extraordinary transmission and narrow-band array antennas.

### References

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