



## Noiseless Control and Amplification of Periodic Waveforms Through Talbot Self-Imaging

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### Abstract

Extracting signals from noise is a fundamental operation in any situation involving the processing and detection of waves, events and information. In particular, periodic signals such as trains of optical pulses and their spectral counterpart, optical frequency combs, are extremely relevant examples of waveforms with deep implications in a myriad of areas of science and technology, where the period of repetition plays a fundamental role in their areas of application. Here, we present a set of lossless linear techniques based on Talbot self-imaging concepts, aimed at re-distributing the energy of periodic temporal and spectral waveforms, achieving an arbitrary control of their period. These techniques can be exploited towards noise mitigation on periodic signals.

### 1. Introduction

It is a common problem to almost any scientific discipline, that the accurate detection of signals, events and information is always limited by the presence of unwanted noise. This is particularly damaging in situations where the signal of interest is weak (i.e., it carries little energy). In such situations, amplification becomes necessary in order to increase the signal energy, however, traditional amplification processes require an active medium and an external supply of energy. This translates into a degradation of the signal-to-noise ratio of the signal of interest, as the active gain process not only amplifies the noise originally accompanying the signal, but also introduces additional noise contributions.

A particularly interesting kind of signals are those that repeat in periodic patterns. In particular, trains of optical waveforms are of great interest in the fields of ultrafast communications and computing, waveform generation, metrology, and nonlinear optics among others [1]. Their spectral counterpart, the optical frequency comb – collections of equally-spaced spectral lines, separated by the free spectral range (FSR)– have become one of the most extended tools in a myriad of areas of science and technology, being the enabling factor of important disciplines such as high-precision spectroscopy and frequency metrology, waveform shaping, and time transfer among others [2]. An adequate control and mitigation of the noise carried by these signals is fundamental to all mentioned fields and disciplines.

Here, we propose a methodology for arbitrarily tailoring the period of repetition of any temporal waveform train or frequency comb, while preserving the entirety of its energy. These methods achieve an unprecedented control in setting the periodicity of repetitive temporal and spectral waves with high precision (a fundamental factor in the areas of application of such signals). More importantly, the proposed methodology relies on linear operations for redistribution of the signal's energy content, achieving an effect of passive amplification that mitigates the impact of undesired noise.

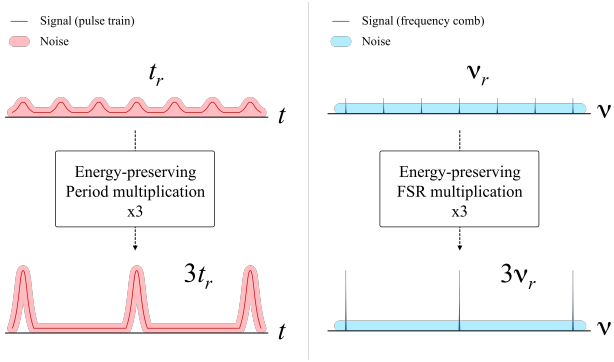
### 2. Methodology

The operation principle of the proposed periodicity control method is based on the theory of temporal and spectral Talbot self-imaging [3]. Temporal Talbot effect is observed when a periodic train of pulses is affected by a very specific quadratic spectral phase (e.g., as introduced by propagation through group-velocity dispersive media), so that the period of said train is divided by a natural number. On the other hand, spectral Talbot effect refers to the Fourier-dual realization of the phenomenon, where a frequency comb is affected by a specific temporal phase (i.e., through a phase modulation mechanism), achieving a division of the original FSR by a natural number.

The relationship between both realizations of the phenomenon, referred to as the time-frequency duality of the Talbot effect, was recently formalized [3]. This consists on a symmetry between the expressions of the aforementioned spectral and temporal phases. A properly engineered combination of such phases can achieve an arbitrary control of the period of any pulse train (pulse period) or frequency comb (FSR). In particular, the period of any given repetitive waveform (either in time or frequency) can be multiplied by any desired integer or fractional factor, higher or lower than 1 [4].

The described methodology is based entirely on linear transformations of the phase of the wave of interest. As such, the complete process preserves the total initial energy of the original wave (aside from practical passive losses associated to an experimental implementation). Moreover, these transformations are coherent, and do not affect the properties of incoherent noise accompanying the signal of interest. This way, the signal energy is redistributed while the noise remains unaltered. If the

mentioned temporal and spectral Talbot phases are designed so that to achieve a period multiplication factor higher than 1, the signal energy per output period is increased, while the noise energy remains unaltered. This leads to an effect of noiseless amplification of the signal of interest, where the impact of said noise is mitigated. Fig. 1 illustrates the impact of these techniques on incoherent noise for a pulse train and a frequency comb whose periods are multiplied by a factor of 3.

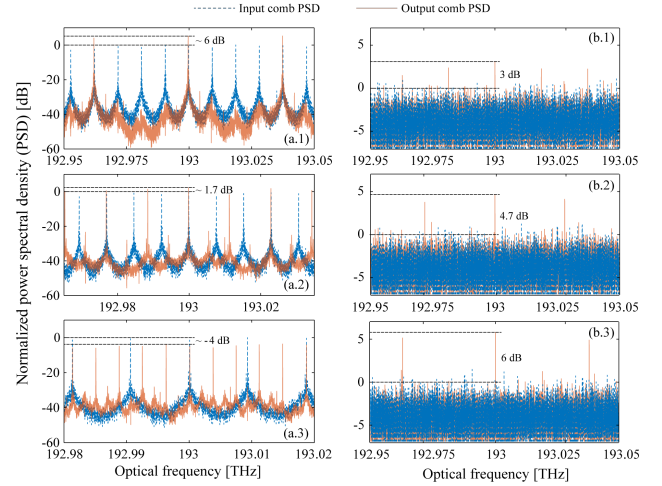


**Figure 1.** Impact of energy-redistribution-based period control on the noise carried by a train of pulses (left,  $t$  stands for time) and a frequency comb (right,  $v$  stands for frequency).

### 3. Results

As an operation example of the proposed methodology for arbitrary period control, here we report experimental multiplication of the FSR of a frequency comb (Fig. 2(a)) by factors of 4 (Fig. 2(a.1), integer FSR multiplication), 1.5 (Fig. 2(a.2), fractional FSR multiplication) and 0.4 (Fig. 2(a.3), fractional FSR division). The predicted conservation of energy ensures that the output comb lines have more (less) peak power when the FSR multiplication factor is higher (lower) than 1. Indeed, the peak line intensity of the output comb is affected by the exact FSR multiplication factor (shown in log scale in Fig. 2).

Moreover, in the extreme case where the signal of interest is completely buried under noise (dashed blue line in Fig. 2(b)), the proposed method is able to recover the signal. The example shown in Fig. 2(b) corresponds to a situation where amplified spontaneous emission noise is injected to the original frequency comb with the purpose of raising its noise floor level to the peak intensity level of the comb lines. Subsequent energy-preserving FSR multiplication of the noisy comb by factors of 2 (Fig. 2(b.1)), 3 (Fig. 2(b.2)) and 4 (Fig. 2(b.3)), increase the energy of the comb lines over the noise floor by 3 dB, 4.7 dB and 6 dB respectively (corresponding to the relative FSR multiplication factor expressed in log scale). It should be noted that in this situation, prior information of the exact absolute frequencies of the comb lines is not necessary for noiseless spectral amplification (only information about the original comb FSR is required by the technique).



**Figure 2.** Demonstration of the period control method on an optical frequency comb. (left) Arbitrary control of the FSR of a frequency comb through energy-preserving transformations. Multiplication factors: 4 (a.1), 1.5 (a.2), 0.4 (a.3). (right) Noiseless spectral amplification of a frequency comb in the situation where the background noise floor completely covers the lines of the comb of interest. Energy preserving FSR multiplication by factors of 2 (b.1), 3 (b.2) and 4 (b.3) produce an effect of amplification of the comb lines over the noise floor by 3 dB, 4.7 dB and 6 dB respectively.

### 4. Conclusion

A general methodology for arbitrary, energy-preserving period control of repetitive temporal and spectral waveforms, based on the time-frequency duality of the Talbot effect was presented. The reported methods have the property of locally reducing the noise content of periodic signals by redistributing the energy carried by the original wave of interest. This feature is an attractive capability with direct implications in the myriad of disciplines that involve the manipulation, processing and measurement of pulse trains and frequency combs.

### 5. References

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