The Propagating Frame with Applications to Local Inverse Scattering

Ram Tuvi and Ehud Heyman
School of Electrical Engineering, Tel Aviv University, Tel Aviv, Israel
ramtuvi@tau.ac.il, heyman@tau.ac.il

Beam summation (BS) formulations are an important approach in wave theory since they provide a ray-based framework for local construction of spectrally uniform solutions in complex configurations. In these formulations, the field is expanded into a phase-space spectrum of collimated beam propagators that emanate from a set of points and directions in the source domain, and thereafter are tracked locally in the medium, and their contributions at the observation point are summed up. In (A. Shlivinski et al., *IEEE Trans. AP*, 52, 2042-2056, 2004) we have introduced an ultra-wide-band phase-space beam summation method (UWB-PS-BS) for radiation from apertures. In this representation the aperture sources are expanded in terms of a discrete set of beams emerging from a set of points and directions in the aperture. The unique feature of this representation is that the beam lattice and the beam propagation parameters are frequency independent so that only one set of beams needs to be calculated and then used for all frequencies. This feature is due to the use of an overcomplete window Fourier transform frames (WFT-F), which adds a degree of freedom that in not available in the conventional Gabor-based expansions.

Recently (M. Leibovich and E. Heyman, *the XXXI General Assembly of URSI*, Beijing, 2014) we have shown that the set of propagating beams constitutes a frame everywhere, i.e., not only in the aperture plane where this set reduces to the conventional WFT-F. In a companion paper (M. Leibovich, R. Tuvi and E. Heyman, *URSI AT-RASC,* 2015) we use the Propagating Frame Theorem to formulate a new expansion scheme for volume source distributions, wherein the source distribution is expanded using the beam set, and the expansion coefficients are obtained by projecting the source distribution onto this beam set. The propagating frame representation provides a new self-consistent framework for wave tracking through weakly scattering media, where the interaction of the propagating beams with the medium heterogeneities is analyzed locally, and then projected onto the same beam set.

In the present paper we apply the UWB propagating frame representation for local UWB tomographic inverse scattering. We describe the interrogating wave as a set of beams and then expand the induced sources, obtained by the interaction of the incident field with the medium heterogeneities, on as a propagating beams frame. The scattering mechanism is thereby described in terms of beam-to-beam scattering coefficients that depend on the local value of the medium heterogeneities. These local heterogeneities may therefore be recovered by backpropagating the scattered beam to the target domain. One of the advantages of this local inverse scattering scheme is that it allows focusing on any desired sub-domains of interest in the target domain, by retaining only those beams that pass through this subdomain.