A General Characterization of Superdirectivity

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This paper presents a general formulation for characterization of any finite antenna as either normal or superdirective. The IEEE Std. 145-2013 defines "Superdirectivity: The condition that occurs when the antenna directivity is significantly higher than that with the array or aperture uniformly excited." This characterization is evidently limited to the special forms of antennas: uniform array or planar aperture antennas. A survey of several popular antenna textbooks shows that, if the topic is listed in the index at all, they cover substantially nothing of a more general nature.

In contrast, the characterization of superdirectivity presented here is entirely independent of antenna structure. It is based instead on fundamental properties of spherical mode fields particularly as elaborated in L. B. Felsen, "Spherical Transmission Line Theory," 1952, and R. F. Harrington, "Time Harmonic Fields," 1961. Questions of impedance match or antenna Q, irrespective of how inextirpable, are secondary consequential effects. TM and TE spherical modes are conventionally distinguished by a radial index n and angular index m. For every mode with radial index n, the mode is essentially cut off in the region $kr < \sqrt{n(n+1)} \approx n$ and propagating in the region $kr > \sqrt{n(n+1)} \approx n$. For an arbitrary finite antenna enclosed by a sphere of minimal radius a, it follows that with normal current excitation within the sphere of radius a modes with index $n > ka$ are filtered out by the cutoff effect or may present among the radiated fields $r > ka$ with only low amplitude.

The maximum directivity achievable by superposition of all modes of radial index $n \le N$ is $(N+1)^2 - 1 \approx N^2$. In view of the cutoff filter effect, the normal directivity of an arbitrary antenna enclosed by a sphere of radius a is constrained by the relation $N \approx ka$ to

$$(ka)^2 = \frac{4\pi}{\lambda^2} \cdot \pi a^2 .$$

We note with some satisfaction that the above constraint agrees with the directivity associated with a uniformly illuminated planar aperture area equal to the cross-section of the sphere of radius a.

An arbitrary antenna structure physically delimited within a sphere of radius a is characterized as superdirective when it attains greater than the normal directivity indicated by the above display equation. This superdirectivity is attainable through greater than normal exciting currents, supercurrents, which overcome the filtering effects of the cutoff spherical region.

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