Uniform asymptotics with respect to the aspect ratio in high-frequency diffraction by an elliptic cylinder

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At high frequencies the scattered field from most bodies is maximal in the direction of incidence. For infinitely thin obstacles such as disks or strips at grazing or near grazing incidence the diffraction picture is different. Evidently that when the body becomes more and more slender, the scattered field can not abruptly change. Taking as an example the case of the elliptic cylinder, we trace the changes in the field of diffraction when the shape of the cylinder varies from a circle to a strip. The asymptotic approach is based on the results of (I. V. Andronov, "Strong elongation versus a large transverse curvature in the problems of high-frequency diffraction", *Antennas and Wireless Propag. Lett.*, **13**, p. 826-827, 2014) and (I. V. Andronov, "High-frequency diffraction by an elliptic cylinder: the near field", *J. of Electromagnetic waves and applications*, **28**, DOI: 10.1080/09205071.2014.966860, 2014). It brings to the uniform expression for the scattering amplitude with respect to the rate of the elongation.

We consider the stationary problem of plane wave diffraction by an infinite perfectly conducting elliptic cylinder with the minor a and the major b semiaxes. The asymptotic decomposition is carried out in inverse powers of the large parameter kb, where k is the wave number of the incident wave. We introduce the parameter $\chi = ka^2/b$ which is considered to be on the order of one, which means that the ellipse in the cross-section of the cylinder is strongly elongated. The near field asymptotics was constructed in the above cited references by means of the parabolic equation method. Substituting these expressions in the Kirchhoff integral we get the asymptotics of the far field amplitude. The leading order term is expressed in the form of the integral, containing the Whittaker functions and depends on the parameter χ and two scaled angles $\alpha = \sqrt{kb}\vartheta_0$ and $\beta = \sqrt{kb}\vartheta$, where ϑ_0 is the angle of incidence and ϑ is the angle of observation. The latter are assumed small, so that α and β are on the order of one.

The asymptotic expression is checked to provide an accurate approximation for the far field amplitude in the case of $kb > 10$. In the final expression one can take infinitely small values of χ, which correspond to the degeneration of the elliptic cylinder to the strip. It is also possible to take large values of χ and even $\chi = kb$, which corresponds to the case of the circular cylinder. This shows that the asymptotics is uniform with respect to the rate of elongation. It allows the variation of the far field amplitude to be traced from the case of the strip, when it is symmetric with respect to the plane of the strip, to the case of circular cylinder when the far field amplitude depends only of the difference $\vartheta - \vartheta_0$.

Some similarity of the effects can be noticed with diffraction by a strongly elongated spheroid, see e.g. (I. V. Andronov, R. Mittra "High-frequency Asymptotics for the Radar Cross-Section Computation of a Prolate Spheroid with High Aspect Ratio", IEEE Transactions on Antennas and Propagation, accepted for publication).