Resonant States in Waveguide Transmission Problems

Yury Shestopalov

Faculty of Engineering and Sustainable Development
University of Gävle , SE-801 76 Gävle, Sweden
yuyshv@hig.se

This work aims at analysis of transmission coefficients of multi-layered dielectric inclusions in a waveguide of rectangular cross-section [1, 2]. We consider the scattering of a normal waveguide mode by layered parallel-plane dielectric diaphragms in a (single-mode) waveguide of rectangular cross section. The closed-form solution to the problem involving explicit formulas for the transmission and reflection coefficients is well known [1, 2]. This enables one to show in particular [2] that the transmission and reflection coefficients considered as functions of one the problem parameter (e.g. permittivity) varying on certain sets are one-to-one mappings and proceed to the solution of inverse problems in different settings [1, 2]. However, one very important feature of this benchmark problem remained untouched, to the best of our knowledge; namely, a fact (known in quantum scattering theory for a similar problem setting, see e.g. [3]) that the transmission and reflection coefficients have singularities in the complex domain of the problem parameters and that the scattering problem under study is not solvable at these singular values of parameters. In this study, we consider the scattering by a one-sectional diaphragm (a 'rectangular barrier' [3]) and prove that in this case the transmission coefficient has singularities in the complex plane of frequency and permittivity and the singularities are associated with resonant states of the transmission problem. We consider in detail the determination of these singularities and propose classification and interpretation of the corresponding solutions. We show in particular that the singularities form an increasing sequence of complex numbers located in a strip, verify their asymptotic behavior, and show that they can be calculated with prescribed accuracy from explicit formulas using the (numerical) solution to simple transcendental equations and that the corresponding solutions have a character of solitons.

1. Y. Shestopalov, Y. Smirnov, Inverse scattering in guides, *J. Phys.: Conf. Ser.*, Vol. 346, 012019, 2012.
2. Y. Shestopalov, Y. Smirnov, and E. Derevyanchuk, Solution to the Inverse Problem of Reconstructing Permittivity of an *n*-Sectional Diaphragm in a Rectangular Waveguide, In *Algebra, Geometry and mathematical Physics*, *Springer Proc. in Mathematics & Statistics, Ser. 10533,* Vol. 85, pp. 555-567, 2014.
3. A. Uma Maheswari, P. Prema and C. S. Shastry, Resonant states and transmission coefficient oscillations for potential wells and barriers, *Am. J. Phys*. 78, pp. 412, 2010.