

## What Did Maxwell Do and How Did He Do It: An Overview of Maxwell's Treatise

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Over a period of about twenty years, Maxwell's determination and unification of the equations of electricity and magnetism evolved from his first paper on the subject in 1855-56, "On Faraday's Lines of Force," to the publication of his *Treatise on Electricity and Magnetism* in 1873. Notwithstanding the many historical accounts and textbooks devoted to Maxwell's work, I have not been able to find a reasonably concise, yet definitive summary of the fundamentals of exactly what Maxwell did in his *Treatise* and how he did it. This talk represents my own attempt to provide such a summary.

I, like countless others, have continually benefited from the depth and rigor of some of his derivations, and, despite some important differences, have been appreciative of how little difference (except for notation) there is between many of his basic definitions and equations and those found in present-day textbooks on electromagnetism. However, I have been especially drawn to some salient features of Maxwell's theory of electricity and magnetism that appear to have been ignored, underemphasized, or possibly overlooked in later textbooks and historical accounts of his work --- at least those with which I am familiar. For example, I, and presumably many others, have been led by commentaries on the *Treatise* to believe that Maxwell did not write down Faraday's law of induction (induced electromotive force) explicitly in his *Treatise* and that it was Heaviside and Hertz who first expressed "Maxwell's equations" in their more familiar contemporary form. Therefore, I was surprised to learn that Maxwell wrote down the integral form of Faraday's law explicitly in his *Treatise* and stated it clearly in words a number of times, even though he did not include either the integral or differential form of Faraday's law explicitly in the summary of his equations in Art. 619. Moreover, he obtained the general form of Faraday's law for moving circuits and this allowed him to derive the electromotive force ( $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ ) per unit moving electric charge and thus the "Lorentz force" equation for moving electric charge.

The talk will also discuss that Maxwell viewed electric charge and current as continua and did not anticipate the discovery that charge comes in discrete units, namely that of the electron. He defined macroscopic continuum fields mathematically in source regions without averaging microscopic fields and related these mathematically defined fields to fields that could be measured in free-space cavities. He used magnetic-charge magnetic dipoles to determine the magnetic field equations so that the  $\mathbf{H}$  field is primary and the  $\mathbf{B}$  field is secondary. We explain how Maxwell determined the magnetic force on current without assuming  $\mathbf{J} = \rho\mathbf{v}$  by modeling current loops as magnetic dipoles and how this led him to believe that the magnetic field would not alter the distribution of existing conduction current; that is, he did not anticipate the Hall effect. Maxwell's summary list of the 12 equations in Art. 619 is rewritten in modern notation. They consist of four main equations (three vector and one scalar), three definitions (two vector and one scalar), three vector constitutive relations, the vector equation for mechanical force density, and the vector magnetic field equation in terms of the gradient of a potential when the magnetic field is irrotational. The talk ends with Maxwell's derivation of electromagnetic wave propagation and the speed of light from his general electromagnetic field equations.