

Twisted Cavity Resonators of Anyon Rotational Symmetry with Electromagnetic Helicity

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Abstract

Twisting the conducting boundary conditions of an electromagnetic cavity with an equilateral triangle cross-section generates eigenmodes with electromagnetic helicity - a result of the coupling of previously degenerate TE and TM modes. This can be interpreted as an emergence of magnetoelectricity, which produces a confirmable shift in resonant mode frequency. By dynamically controlling the twist angle of a cavity resonator, we demonstrate real-time, macroscopic manipulation of electromagnetic helicity and mode frequency.

1. Introduction

The differential interaction of helical radiation with chiral and non-chiral materials has garnered significant interest in fields such as materials science, nanophotonics, and quantum information processing, with applications in molecular chirality detection, advanced communication technologies, and dark matter research [1].

Electromagnetic helicity is intrinsically related to the dual transformation of the electromagnetic state, which effectively rotates the electric and magnetic properties of the medium that induces magnetoelectric coupling. Mathematically, electromagnetic helicity can be quantified by the following normalised parameter:

$$h_i = 2\text{Im} \left[\vec{e}_i(\vec{r}) \cdot \vec{h}_i^*(\vec{r}) \right] \quad (1)$$

$$= \frac{2\text{Im}[\vec{E}_i(\vec{r}) \cdot \vec{H}_i^*(\vec{r})]}{V_i E H},$$

where E and H are real constants, $\vec{E}_i(\vec{r}) = E\vec{e}(\vec{r})$ and $\vec{H}_i(\vec{r}) = H\vec{h}(\vec{r})$ are the time-dependent electric and magnetic vector fields of some mode i , respectively, and $\vec{e}(\vec{r})$ and $\vec{h}(\vec{r})$ are the normalised position dependent eigenvectors such that $\int \vec{e}(\vec{r})^* \cdot \vec{e}(\vec{r}) dV = \int \vec{h}(\vec{r})^* \cdot \vec{h}(\vec{r}) dV = 1$. Thus, for a resonant mode in some cavity, the total helicity of a given mode i , $\mathcal{H}_i$, may be written as the integral of (1) over the cavity volume:

$$\mathcal{H}_i = \int h_i dV. \quad (2)$$

It has been thought that an imaginary non-zero value of $\vec{E}_i(\vec{r}) \cdot \vec{H}_i^*(\vec{r})$ could not be generated in a cavity resonator [2]. However, in this work, we present evidence of resonant electromagnetic cavity modes that exhibit non-zero helicity *in vacuo*. By introducing an arbitrary twist angle around the cavity's central axis, we break the mirror symmetry of the resonator, thereby creating a chiral geometry.

Simulating the twisting of an electromagnetic cavity resonator with a triangular cross-section using finite element modelling (FEM) software reveals that breaking this mirror symmetry results in the magneto-electric coupling of near-degenerate TM_{11p} and $\text{TE}_{11(p+1)}$ modes. This results in the generation of a new orthogonality basis for the twisted resonator, appearing as either in-phase mixing of the form $\text{TM}_{11p} + \text{TE}_{11(p+1)}$ (labelled ψ_{p+1}^+) or out-of-phase mixing of the form $\text{TM}_{11p} - \text{TE}_{11(p+1)}$ (ψ_{p+1}^-).

The mode mixing process can be understood as normal mode splitting, and results in frequency tuning of the $\psi^\pm$ modes as a function of ϕ . The left-handed and right-handed nature of these helical modes are demonstrated in Figure 1.

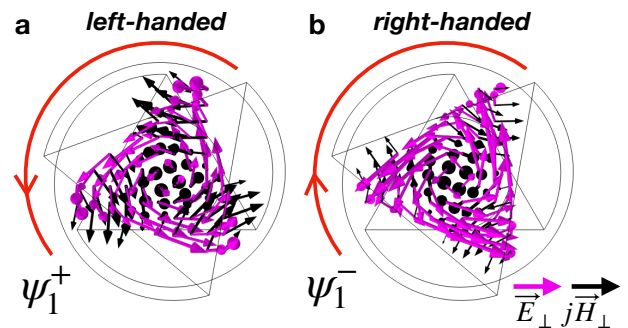


Figure 1. The transverse electric field, $\vec{E}_\perp$ (magenta), and the transverse magnetic field, $j\vec{H}_\perp$ (black), in the triangular cross-section twisted resonator with a twist angle of $\phi = \frac{10\pi}{9}$. Results are shown for (a) the ψ_1^+ mode and (d) the ψ_1^- mode.

The twisting process can be interpreted as introducing a so-called material "chirality parameter" into the propagation media, where an empirical relationship between κ and ϕ

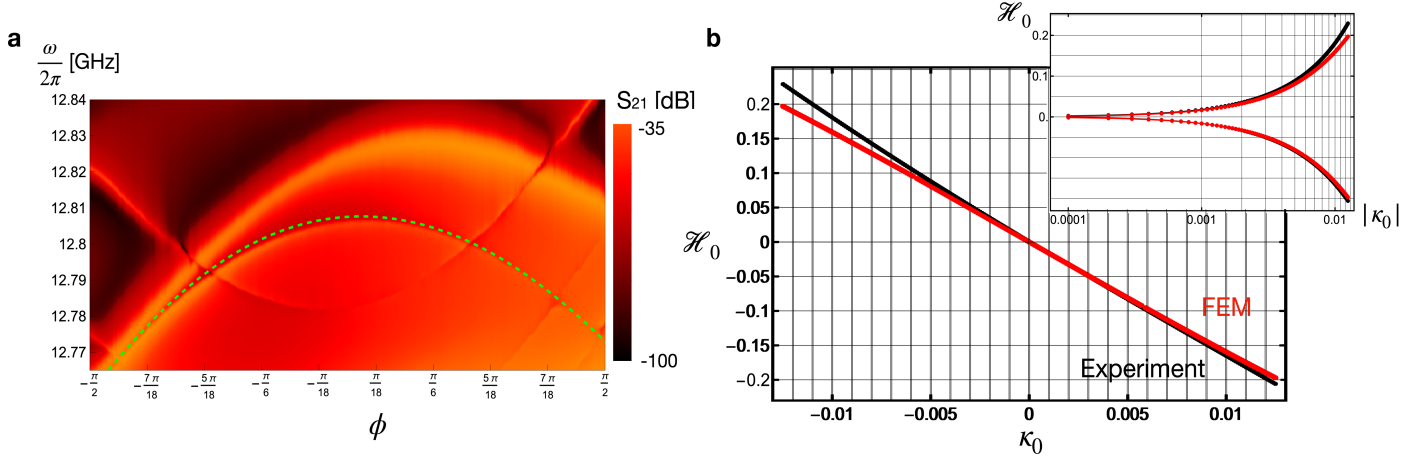


Figure 2 (a) Transmission spectrum measured from the WR137 rectangular cavity resonator (35.48 mm by 16.1 mm) of length $l = 312.3$ mm as a twist ϕ is introduced to the cavity. The frequency of the mode resulting from the mixing of $TE_{20(20)}$ and $TE_{11(16)}$ is tracked by the yellow dashed line. (b) The experimentally derived $\mathcal{H}_0$ using (3) are shown in black as a function of the empirically derived κ for resonator twist- ϕ . Alongside this is the $\mathcal{H}_0$ values derived from FEM simulated eigenmodes of the untwisted WR137 rectangular cavity resonator with some effective κ introduced to the propagation media, marked by the red curve. The subplot displays this data on a log-linear scale.

can be derived for a given cavity mode. From perturbation theory [2], a fundamental relationship can be derived between the helicity $\mathcal{H}_i$ and the frequency dependence on κ of a resonant mode, $\delta\omega/\delta\kappa = \omega_1 - \omega_0/\kappa_1 - \kappa_0$, expressed as:

$$\left(\frac{\delta\omega}{\delta\kappa \omega_0} \right)_{\kappa_0 \ll 1} = \frac{\mathcal{H}_i}{2\mu_r \epsilon_r}, \quad (3).$$

where ω_i is the unperturbed cavity frequency, μ_r and ϵ_r the media's relative permeability and permittivity, respectively.

Frequency spectral response measurements of 3D-printed cavities with $\phi = 0$ and $\phi = \pm 10\pi/9$ confirm the predicted frequency shifts from FEM and (3), indicating the generation of electromagnetic helicity in a hollow, free-space volume.

By physically twisting an electromagnetic cavity resonator with a WR137 rectangular cross-section and length $l = 312.3$ mm, we demonstrate the dynamic frequency tuning of non-zero helicity modes in real time. This twistable electromagnetic resonator was constructed using a commercially available flexible waveguide that had its ends capped, one of which was attached to a rotary stage.

The frequency of the non-zero helicity mode generated due to a mixing of the $TE_{20(20)}$ mode and the $TE_{11(16)}$ mode is tracked as a function of ϕ by the yellow-dashed curve on the transmission spectra in Figure 2(a). According to (3), this dynamic frequency tuning implies that helicity is also being varied in real-time with twist angle.

The perturbation-derived $\mathcal{H}_0$ from experiment and (3) is plotted in black in Figure 2(b) as a function of the empirically derived κ for resonator twist- ϕ . This is compared to $\mathcal{H}_0$ derived from simulation using (2), as shown in red. There is good agreement between simulated and experimental results in the $\kappa \ll 1$ regime.

We have demonstrated experimentally that twisting an electromagnetic resonator enables real-time modulation of helicity and resonant frequency. The twisting mechanism introduces magneto-electric coupling that mixes electric and magnetic fields to generate chiral electromagnetic radiation in a monochromatic resonant mode in vacuo. Our approach provides a tunable platform for helicity control, with potential applications in material chirality detection, encryption, and communication technologies.

6. Acknowledgements

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References

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