

Fault Location in Multi-Conductor Transmission Lines Using Electromagnetic Time Reversal: The Extended Bounded Phase Property

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Abstract

Electromagnetic Time Reversal (EMTR) features the bounded phase property, a well-established and reliable method for fault location in two-conductor transmission lines. Under the EMTR-based framework for mismatched media, this property asserts that the phase of the overall forward and backward transfer function remains strictly confined at the true fault location. Extending this property to multi-conductor lines is challenging because the transfer function ceases to be a scalar and instead becomes a matrix. Nonetheless, in this paper, we demonstrate that the bounded phase property applies to all eigenvalues of the transfer-function matrix, thereby providing a clear pathway for its application to multi-conductor lines.

1 Introduction

Electromagnetic time reversal (EMTR) exploits the inherent time-reversal invariance of Maxwell's equations, specifically the wave equations underlying electromagnetic field propagation [1]. In a fundamental EMTR-based procedure for localising an excitation source (such as a fault) on transmission lines, a transient signal generated by the source is measured at an observation point. This measured signal is then reversed in time (or, equivalently, complex-conjugated in the frequency domain) and re-injected into a model of the same medium. Ideally, if the system is lossless and the model is accurate, the wave refocuses at the origin of the transient event [2, 3]. In practical transmission lines, however, time-reversal invariance does not strictly hold due to inherent losses. Nonetheless, EMTR techniques remain highly effective for transient event localisation by approximating time invariance while benefiting from accurate modelling of wave propagation and reflection phenomena [4].

EMTR in mismatched media occurs when the medium during backward (reverse time) propagation differs from that in the forward propagation. A purpose-built case that exemplifies this is when the unknown shunt branch (representing the short-circuit path) is absent in the backward model, as schematically illustrated in Figs. 1a and 1b [5, 6]. A key advancement in source localisation techniques for

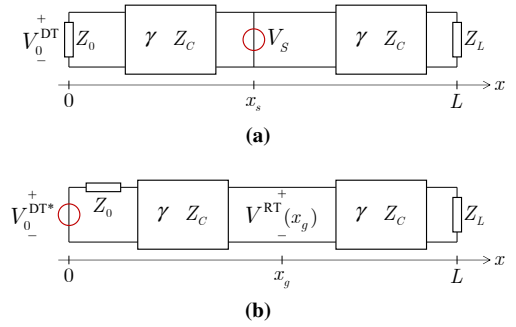


Figure 1. Two-port representation of the EMTR in mismatched media based fault location procedure: (a) forward propagation and (b) backward propagation.

such a lumped mismatched-media condition is the introduction of the *bounded phase property* [5]. In the context of two-conductor transmission lines (single-phase overhead lines or underground cables, for instance), eliminating the shunt branch during backward propagation produces a distinct signature at the source location in the frequency domain. Specifically, if the original transient is a step-like excitation (as is typical for a sudden fault), the phase of the backwards-propagated voltage at the source location remains confined within a fixed interval. This characteristic is referred to as the bounded phase property.

Although substantial progress has been achieved, most prior studies on the bounded phase property have primarily concentrated on two-conductor transmission lines, either incorporating realistic losses or accounting for topological complexity. In this paper, we extend the property to multi-conductor transmission lines and demonstrate that the fundamental principle remains robust even in the presence of inter-conductor coupling.

2 Bounded Phase Property in Two-Conductor Transmission Lines

The bounded phase property was established through the application of EMTR in lumped mismatched media for fault location along transmission lines [5]. In the forward-propagation (direct-time) scenario, as depicted in Fig. 1a, the two-port network incorporates the unknown fault

branch at $x = x_s$. Let $V_0^{\text{DT}}(j\omega)$ be the voltage measured at the observation point, $x = 0$, when the fault is simulated by a source $V_S(j\omega)$. The *forward transfer function* $H^{\text{DT}}(j\omega)$ is defined as the relationship between $V_0^{\text{DT}}(j\omega)$ and $V_S(j\omega)$, expressed as

$$V_0^{\text{DT}}(j\omega) = H^{\text{DT}}(j\omega)V_S(j\omega). \quad (1)$$

Figure 1b illustrates the backward-propagation (reverse-time) scenario, where the line model intentionally excludes the unknown fault branch. In this case, the time-reversed version of the measured signal, denoted as $V_0^{\text{DT}*}(j\omega)$ (where ‘*’ indicates the complex-conjugate operation), is re-injected into this mismatched network from the same observation point. Let $V^{\text{RT}}(x_g, j\omega)$ represent the resulting voltage at a general line position, $x = x_g$. The *backward transfer function*, $H^{\text{RT}}(x_g, j\omega)$, is then defined through

$$V^{\text{RT}}(x_g, j\omega) = H^{\text{RT}}(x_g, j\omega)V_0^{\text{DT}*}(j\omega). \quad (2)$$

The bounded phase property asserts that, for a step-like fault excitation, the backwards-propagated voltage at the true fault location, $V^{\text{RT}}(x_g = x_s, j\omega)$, maintains its argument (phase) strictly within the $0\text{--}\pi$ interval over the entire frequency spectrum. However, any misalignment (i.e., any guessed location not coincident with the true fault location) yields phase values crossing outside that bound at certain (usually higher) frequencies. Consequently, merely observing whether the phase deviates from the $0\text{--}\pi$ interval allows one to identify the actual fault position unambiguously.

Note that, owing to the step-like nature of the fault source, characterised by $V_S(j\omega) = 1/j\omega$, an inherent $-\pi/2$ phase shift is introduced. Consequently, when the *overall transfer function* (combining both forward- and backward-propagation processes) is defined as

$$H(x_g, j\omega) = H^{\text{RT}}(x_g, j\omega)H^{\text{DT}*}(j\omega), \quad (3)$$

the bounded phase property is essentially preserved by the transfer function, with only a shifted phase relative to the backwards-propagated voltage. Concretely, rather than remaining within $[0, \pi]$, $H(x_g, j\omega)$ is strictly and exclusively confined within the interval $[-\pi/2, \pi/2]$ at the true fault location.

For illustration purposes, consider a numerical example of a two-conductor transmission line characterised by a per-unit-length (p.u.l.) inductance $l' = 1.1 \mu\text{H/m}$, a p.u.l. capacitance $c' = 10.7 \text{ pF/m}$, and a p.u.l. resistance $r' = 0.268 \text{ m}\Omega/\text{m}$. The line is terminated at both ends by power transformers, each modelled as a high-impedance load of $Z_0 = Z_L = 10 \text{ k}\Omega$. Assume a total line length of 10 km, with a fault occurring at 3.5 km. The bounded phase property can be visualised by plotting the complex values of the transfer function $H(x_g, j\omega)$ in the complex plane, as shown in Fig. 2. By examining these plots, the distribution of the components of $H(x_g, j\omega)$ across specific quadrants can

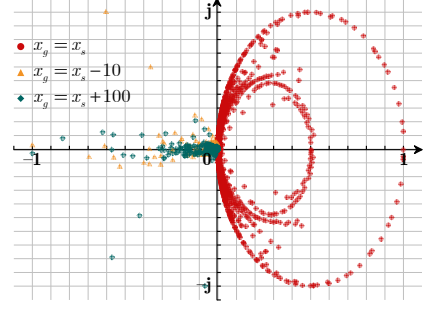


Figure 2. Four-quadrant normalised distribution of $H(x_g, j\omega)$. For cases where $\Delta x(\text{m}) = x_g - x_s \neq 0$ (in meters), only the outlier components that appear in Quadrants II and III are plotted.

be analysed, providing an insight into the phase behaviour. When the observation point x_g coincides with the fault location x_s , the argument of $H(x_g = x_s, j\omega)$ remains strictly confined within $\pm\pi/2$, as evidenced by the fact that all components of $H(x_g = x_s, j\omega)$ lie exclusively within Quadrants I and IV. By contrast, if there is a longitudinal offset $\Delta x = x_g - x_s \neq 0$, some components of $H(x_g = x_s + \Delta x, j\omega)$ migrate into Quadrants II and III, illustrating that the phase bound has been violated.

3 Extension of Bounded Phase Property in Multi-Conductor Transmission Lines

We summarise the extension of the bounded phase property to multi-conductor systems by presenting results from a typical three-phase setup with four conductors (e.g., three phases plus a neutral or ground conductor). In this scenario, we analyse a balanced fault to establish a direct analogue to the two-conductor setting of Fig. 1a. Specifically, the source term, originally represented by a single complex scalar per frequency, is now expanded into a three-by-one vector $\mathbf{V}_S(j\omega)$ for each frequency, indicating identical fault excitations on each phase conductor. That is, $\mathbf{V}_S(j\omega) = [V_{S,i}(j\omega)]_{1 \leq i \leq 3}$, with $V_{S,i}(j\omega) = 1/j\omega$.

Even under balanced excitation and terminations, merely tracking the phase voltages,

$$\mathbf{V}^{\text{RT}}(x_g, j\omega) = [\mathbf{V}_i^{\text{RT}}(x_g, j\omega)]_{1 \leq i \leq 3}, \quad (4)$$

leads to the invalidation of the bounded phase property. Specifically, none of the phase voltages remain strictly confined within a constant interval across all frequencies. Moreover, a second aspect of the breakdown of the original bounded phase property is that for multi-conductor lines, the overall transfer function at any given frequency and location,

$$\mathbf{H}(x_g, j\omega) = \mathbf{H}^{\text{RT}}(x_g, j\omega)\mathbf{H}^{\text{DT}*}(j\omega), \quad (5)$$

is no longer a single complex number but a matrix with complex elements. Consequently, the inapplicability of phase definitions to matrices prevents the investigation of

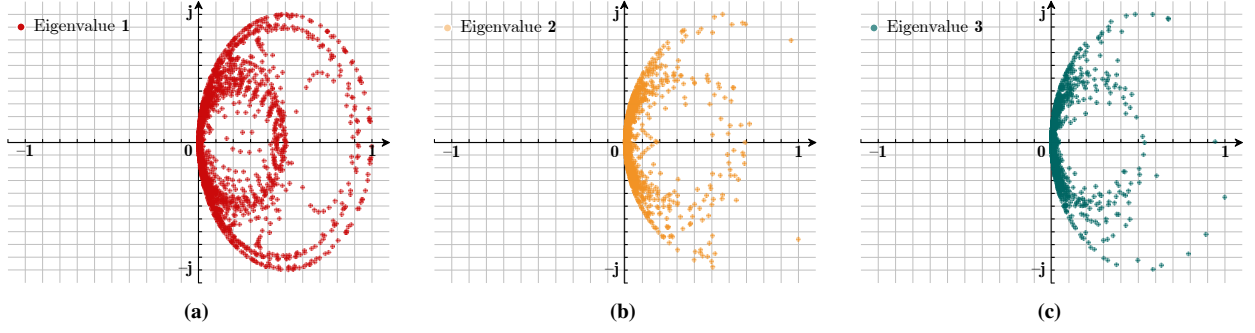


Figure 3. Four-quadrant normalised distribution of the eigenvalues of $\mathbf{H}(x_g, j\omega)$ for the case $x_g = x_s$.

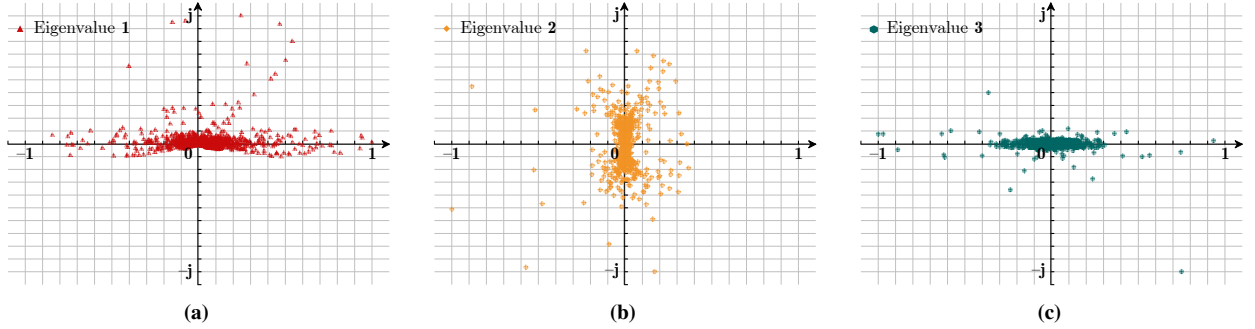


Figure 4. Four-quadrant normalised distribution of the eigenvalues of $\mathbf{H}(x_g, j\omega)$ for: (a) Eigenvalue 1 for the case $x_g = x_s - 10$ m, (b) Eigenvalue 2 for the case $x_g = x_s + 100$ m, and (c) Eigenvalue 3 for the case $x_g = x_s + 2500$ m.

the bounded phase property through transfer functions, as would be possible in the context of two-conductor lines. Instead, multi-conductor systems necessitate the consideration of more advanced matrix attributes to detect the phase constraints that become relevant for fault-location purposes.

In simpler terms, the matrix nature of the overall transfer function does not outright eliminate the possibility of the bounded phase property; rather, it relocates the phenomenon to the domain of eigenvalues. Each eigenvalue of the overall transfer function matrix remains a complex scalar with a well-defined argument that can be analysed across frequencies. Notably, at the true fault location, these eigenvalues exhibit phase arguments constrained within the specific interval $\pm\pi/2$, precisely aligning with the characteristic bounds previously established for two-conductor lines. In other words, the phenomenon originally characterised as a single bounded phase of $H(x_g, j\omega)$ in the two-conductor setting is reinterpreted in multi-conductor systems, where the overall transfer function matrix $\mathbf{H}(x_g, j\omega)$, serving as the counterpart of $H(x_g, j\omega)$, collectively exhibits this property through its eigenvalues.

Using the three-phase system parameters in [4], numerical experiments confirm that the bounded phase property holds when examining the eigenvalues of the overall transfer function $\mathbf{H}(x_g, j\omega)$. Fig. 3 shows that, at the true fault location, $x_g = x_s$, all three eigenvalues are strictly confined to Quadrants I and IV, adhering to the $\pm\pi/2$ phase restriction. Conversely, at the three non-faulted locations (see Fig. 4), none of the three eigenvalue distributions remain fully

confined to those quadrants, with each eventually extending into Quadrants II or III for at least some frequencies.

One way to view the eigenvalues of the overall transfer function matrix is to consider each eigenvalue as defining a separate mode [7], each with its own magnitude and phase behaviour across frequencies. Just as a two-conductor transfer function possesses a single argument that varies with frequency, each eigenvalue in the multi-conductor scenario traces its own trajectory in the complex plane. Consequently, although the system's response is governed by an overall matrix, analysing the eigenvalues restores a complex scalar-like representation: each mode can exhibit a bounded argument in the same manner as a two-conductor transfer function.

4 Conclusion

We presented an illustration of the extension of the EMTR-based bounded phase property from two-conductor transmission lines to multi-conductor scenarios. While the bounded phase property does not directly apply to either the backwards-propagated voltages or the matrix-structured overall transfer functions, we demonstrated that each eigenvalue of the transfer function matrix remains confined within $\pm\pi/2$ exclusively at the true fault location. This eigenvalue-centric perspective preserves the effectiveness of EMTR-based fault location within the lumped mismatched-media framework.

5 Acknowledgements

This work was supported by the Excellent Young Scientists Fund Program of the National Natural Science Foundation of China (NSFC).

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