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Efficient Direction of Arrival Estimation in Indoor Environments using a Sparse Fourier Transform-Based Split-Step Parabolic Equation Method

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Abstract

The direction information of the electromagnetic waves plays an important role in the deployment of 5G and beyond wireless communication networks. This necessitates the research on numerical methods to estimate the direction of arrival (DoA) information in such systems. The parabolic equation (PE)-based DoA estimation method has emerged as a promising technique due to its robustness and stability. However, the computational cost of the PE-based methods increases significantly as the frequency rises. This paper applies a sparse Fourier transform technique to accelerate the DoA estimation process. The performance is investigated in an indoor corridor and validated against theoretical results.

1 Introduction

In recent years, the use of direction information has been considered in various fields including wireless communication systems, localization, and autonomous vehicles [1, 2, 3]. There has been significant interest in research on the development of DoA estimation methods. The parabolic equation (PE) methods [4, 5] have been successfully employed in the past to obtain reliable channel modeling of radio wave propagation in large-scale environments. The simultaneous calculation of received signal strength at numerous sampling points makes PE a natural choice for DoA estimation. However, despite the proven promise of PE-based DoA estimation methods, the rapidly growing computation time at higher frequencies poses a challenge [6]. This limitation renders the approach unable to meet the demands of emerging real-time applications in various scenarios.

In this paper, a spare Fourier transform technique [7, 8] is introduced to reduce the computational cost involved in the process of wave propagation modeling. The generated signal strength is subsequently employed for calculating the DoA information using the classical multiple signal classification (MUSIC) algorithm. By calculating only a subset of Fourier coefficients instead of applying the Fourier transform across the entire two-dimensional transverse plane, a significant reduction in computational time is achieved. The proposed method and the conventional PE-based DoA estimation methods are demonstrated in an indoor corridor example and validation against analytical results is also given.

2 The Proposed DoA Estimation Method

2.1 The Split-Step Parabolic Equation Methodethod

The PE method is derived from the Helmholtz wave equation for free space:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k_0^2\right)\phi(x,y,z)=0 \tag{1}$$

where ϕ is a scalar potential and k_0 is the free-space wave number. In this paper, assuming propagation along the z-axis, a solution to (1) can be cast into the form:

$$\phi(x,y,z)=u(x,y,z)\,\mathrm{e}^{-jk_0 z} \tag{2}$$

where u is the reduced plane wave solution. Assuming that [4]:

$$\left|\frac{\partial u^2}{\partial z^2}\right| \ll k_0\left|\frac{\partial u}{\partial z}\right|. \tag{3}$$

The standard parabolic equation can be expressed as:

$$\frac{\partial u}{\partial z}=\frac{1}{2jk_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u \tag{4}$$

where u is the reduced plane wave solution. A commonly used split-step Fourier technique [9] can be utilized for its numerical implementation, as follows:

$$u(x,y,z+\Delta z)=F^{-1}\left\{C(k_x)\,C(k_y)\,F\left\{u(x,y,z)\right\}\right\} \tag{5}$$

where u is the reduced plane wave solution; F and F^{-1} are Fourier transform pairs; k_x and k_y are spectral variables, and

$$C(k)=\exp\left(\frac{-jk^2\Delta z}{2k_0}\right). \tag{6}$$

All field components can be determined iteratively using (5).

2.2 Proposed DoA Estimation Method

Our proposed fast DoA estimation algorithm combines the principles of PE methods with sparse matrix techniques, resulting in substantial improvements in processing speed. An overview of the method is shown in Fig. 1, with a detailed breakdown of the procedure provided below:

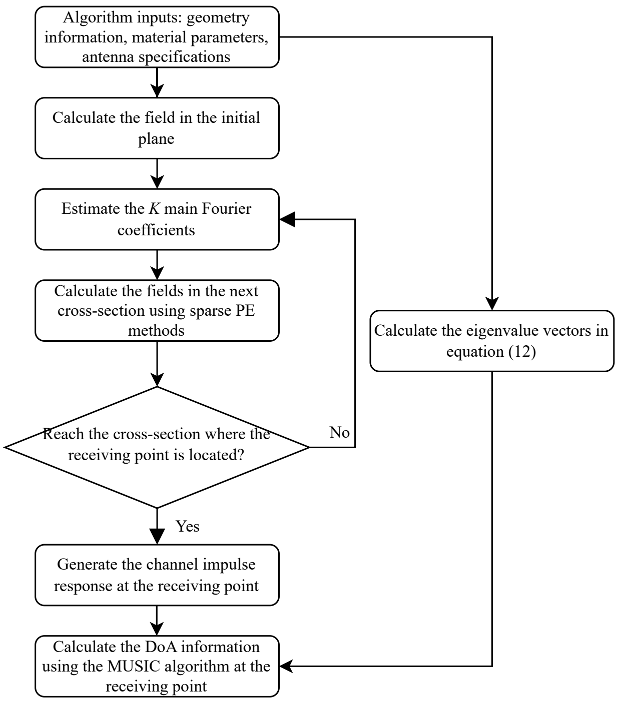


Figure 1. Diagram of the proposed DoA estimation algorithm.

Step 1: Generate the $N \times N$-dimensional fields in the initial plane through the antenna specifications information.

Step 2: Estimate the K main Fourier coefficients.

Step 3: Use (5) to generate the $N \times N$-dimensional fields in the next cross-section.

Step 4: Repeat Steps 1-3 until reach the cross-section where the receiving point is located.

Step 5: Calculate the corresponding eigenvalue vectors in (12).

Step 6: Generate the channel impulse response information at the receiving point.

Step 7: Estimate the DoA information at the receiving point.

In Step 3, the field update procedure for each cross-section is applied iteratively until the propagator reaches the desired range. In this work, we group the Fourier coefficients into a limited number of categories, referred to as baskets, as described in 2.2.1 [7, 8]. Each basket is expected to contain only one dominant coefficient, meaning that baskets without large coefficients can be discarded to reduce computational costs.

Furthermore, the locations of the large coefficients, which are necessary for reconstructing the entire matrix, still need be determined. To achieve this, we introduce the Hash function to estimate the positions of these large coefficients, as detailed in 2.2.2.

2.2.1 Binning the Fourier Coefficients

We provide a brief introduction to the binning method employed in this work. Initially, the signal is permuted to isolate the large coefficients [10]:

$$\mathbf{P}(x,y,z) = \mathbf{U}(\sigma_1 x + \tau_1, \sigma_2 y + \tau_2, z). \tag{7}$$

$\mathbf{P}$ denotes the signal after permutation, while σ_1, σ_2, τ_1, and τ_2 are random integers. Next, a subsampling process is applied to extract the large coefficients and reduce the signal data dimensionality:

$$\mathbf{Z}_z(i,j) = \sum_{u=0}^{N/B-1} \sum_{v=0}^{N/B-1} \mathbf{P}\left(i+Bu, j+Bv, z\right) \tag{8}$$

where $i, j \in [1, B]$. Following subsampling, the matrix $\mathbf{Z}_z$ is compressed to a $B \times B$ size, with the primary Fourier coefficients distributed into $B \times B$ baskets at the z-th cross-section.

2.2.2 Estimation of Main Fourier Coefficients

Using the Hash function, a set of coordinates I can be obtained for all large coefficients in the $B \times B$ baskets.

$$h_{\sigma_1,\sigma_2}(i,j) = \mathrm{round}(\sigma_1 \sigma_2 i j \frac{B^2}{N^2}). \tag{9}$$

Multiple sets $I_1, I_2, \ldots$ are generated, and the positions of the estimated Fourier coefficients can also be obtained.

2.2.3 DoA Estimation

After applying the Fourier transform technique, the baseband impulse response of the channel can be derived through an inverse Fourier transform applied to the frequency-domain electromagnetic field distribution:

$$h(x,y,z,t_m) = N\Delta f \sum_{n=0}^{N-1} H(\omega_n, x, y, z) \exp(\frac{i2\pi mn}{N}), \tag{10}$$

where N denotes the number of selected frequency points, and Δf is the frequency interval. H is the field distribution of the receiver at different frequency points ω_n, and $t_m = z/c + m * \Delta t$ is the propagation time. Δt denotes the time-domain resolution, and z is the projection distance between the transmitter and the receiver.

To generate the DoA information, the classical MUSIC algorithm is employed. A uniform linear array with I elements is assumed to be placed at the receiving location. The impulse response calculated in (10) can be used to construct the input matrix which contains the time-domain received information at different points:

$$A = XX^H = \begin{bmatrix} h(x,y,z,t) \\ h(x,y,z+\Delta z,t) \\ \cdot \\ \cdot \\ \cdot \\ h(x,y,z+(i-1)\Delta z,t) \end{bmatrix} \begin{bmatrix} h(x,y,z,t) \\ h(x,y,z+\Delta z,t) \\ \cdot \\ \cdot \\ \cdot \\ h(x,y,z+(i-1)\Delta z,t) \end{bmatrix}^H \tag{11}$$

Subsequently, an eigenvalue decomposition can be applied to matrix A, yielding a total of I eigenvalues and their corresponding eigenvectors. Let D represent the number of paths of incoming waves. A vector E_n can be formed by selecting the $I-D$ smallest eigenvalues, which are regarded as unrelated to the incoming waves. Another vector, denoted as $a(\theta) = [1, e^{-j\frac{2\pi}{\lambda}\Delta z \sin\theta}, \ldots, e^{-j\frac{2\pi}{\lambda}(I-1)\Delta z \sin\theta}]$, is considered orthogonal to E_n and is defined based on the assumed array structure. Then we can get:

$$a^H(\theta)E_n = 0 \quad \text{for } \theta = \theta_1, .., \theta_D \tag{12}$$

where $\theta_1 \sim \theta_D$ denotes the DoA (in degree). The direction information can be found as [11, 12]:

$$P_{\text{MUSIC}}(\theta) = \frac{1}{\left| a^H(\theta) E_n {E_n}^H a^H(\theta) \right|}. \tag{13}$$

This results in peaks in the spectrum at the signal directions, stemming from the orthogonal relationship between E_n and $a(\theta)$, and DoA information can be determined by identifying the peaks in P_{MUSIC}.

3 Results in an Indoor Corridor

For validation purposes, the conventional PE-based DoA estimation methods and the proposed methods are applied in a rectangular indoor corridor with perfectly conducting walls. The corridor cross-section's dimensions are $4\text{m} \times 4\text{m}$, and the longitudinal length of the corridor is 100m, as shown in Fig. 2. The relative dielectric permittivity and conductivity of the surrounding walls are set as $\varepsilon_r = 5$ and $\sigma_0 = 0.01$ S/m, respectively. The center frequency is set as 4.9 GHz. A unit-strength Gaussian source with a standard deviation of 3 dB is placed at the center of the transverse plane, and the receiver is placed at the same position along the corridor. The frequency interval and the number of selected frequency points in (10) are set as 100 MHz and 91, respectively. The simulation time of the conventional PE-based DoA estimation and the proposed methods are 52.8 s and 9.3 s, respectively. By employing the conventional SSPE methods and the proposed methods, the spatial distribution of the electric field on the cross-section at a distance of 100 m is illustrated in Fig. 3. The results of DoA estimation at various delay values are summarized in Fig. 4 and Table 1. We can see that the predicted results of the two methods both have good agreement with the theoretical results. However, the proposed model can achieve significant computational savings.

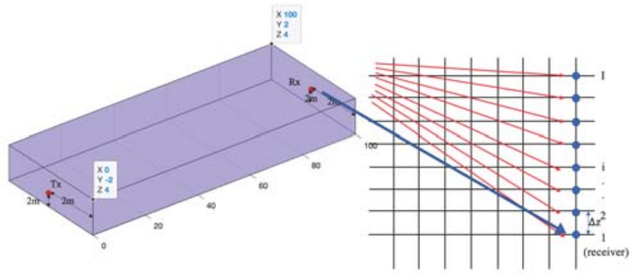


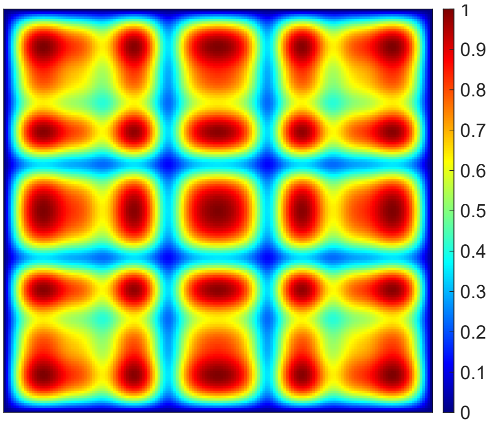
Figure 2. Diagram of the considered indoor corridor.

Table 1. DoA results generated by the conventional PE-based and the proposed methods in the considered indoor corridor.

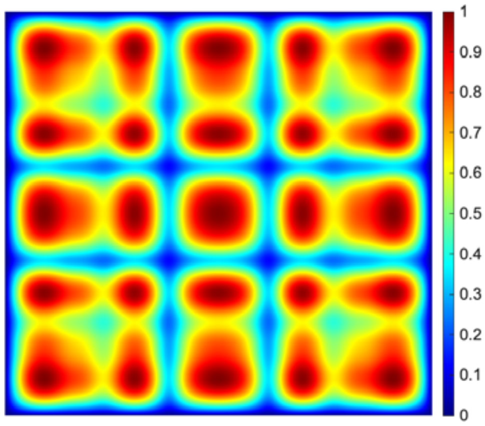
Path Number	Theoretical Results [degree]	The Conventional Method	The Proposed Method
Path 1	0	0	0
Path 2	2.29	2.48	2.34
Path 3	4.57	4.47	4.72
Path 4	6.84	7.02	7.02
Path 5	9.09	9.54	9.33

4 Conclusion

This paper presents an efficient DoA estimation method for indoor environments, leveraging a sparse Fourier transform-based SSPE technique. The proposed method significantly reduces computational costs compared to conventional PE-based approaches while maintaining high accuracy. By employing sparse matrix techniques and selective Fourier coefficient estimation, the method accelerates the wave propagation modeling process, making it suitable for real-time applications in high-frequency scenarios. Future work will focus on further optimizing the algorithm for even lower computational complexity and extending its application to more complex indoor environments, such as those with irregular geometries. This research aims to contribute to the development of advanced wireless communication systems, particularly for 5G and beyond technologies, by providing a reliable and efficient solution for direction information acquisition.



(a)



(b)

Figure 3. The spatial distribution of the electric field on the cross-section at a distance of 100 m from the source plane $z = 0$ in the considered indoor corridor. (a) conventional SSPE method. (b) sparse SSPE method.

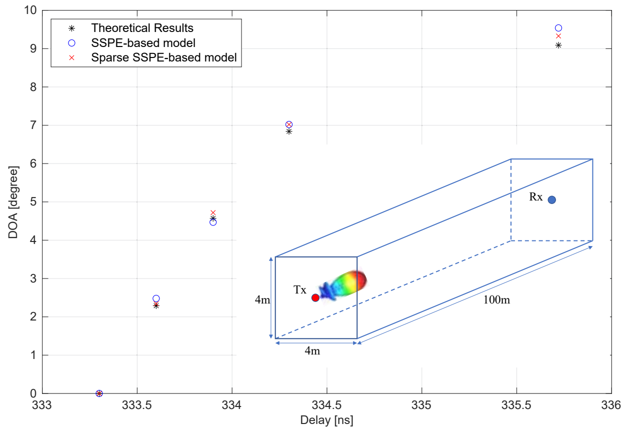


Figure 4. Comparison of the DoA with the conventional PE-based and the proposed methods in the considered indoor corridor.

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