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Thermal Emission Effects in Time-Varying Dispersive Media

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Abstract

In the last few years, temporal metamaterials have emerged as a powerful platform for dynamically controlling optical properties and light-matter interactions, becoming one of the most active research areas in optics and nanophotonics. This concept has recently been extended to thermal emission, establishing a theoretical framework for incandescence in time-varying media and showing extraordinary thermal emission effects. However, for simplicity, that analysis has been restricted to non-dispersive (instantaneous) time modulations. In this work, we extend this framework to include dispersion effects, a fundamental aspect gaining increasing attention in the field. The main contribution is the identification of a non-physical divergence at high frequencies in the emission spectra – a feature attributed to the non-dispersive approach–which is resolved by incorporating dispersion, naturally leading to high-frequency spectral cutoffs. Beyond enabling more accurate emission spectra and the possibility to calculate total radiated power calculations, our findings provide key theoretical insights that could aid experimental research on thermal emission in time-varying media.

1 Introduction

Recent progress in nanophotonics and materials science has enabled advanced *thermal emission engineering* [1]. Just like nanophotonic engineering, the main goal of thermal emission engineering is controlling and enhancing the optical coherence properties of thermal fields, ultimately related with the spectral bandwidth (temporal coherence), directivity (spatial coherence), and degree of polarization (polarization coherence). So far, most of the practical implementations to this aim have been relied on passive approaches based on structuring matter and shaping their optical properties, i.e., on the use of photonic nanostructures. Yet, in the last few years, active approaches for dynamic control of thermal emission are increasingly gaining interest [2]. Building upon these possibilities, the emerging field of *temporal metamaterials* [3] (often simply referred to as time-varying or time-modulated media) has motivated the introduction of *incandescent temporal metamaterials* [4]. From a theoretical framework based on *macroscopic quantum electrodynamics* (QED), it has been predicted a number of extraordinary thermal emission effects from time-modulated media, including nonlocality-induced enhancement of spatial coherence, super-Planckian far-field emission (i.e., thermal emission exceeding the limit of the blackbody spectrum), and vacuum amplification effects at finite temperatures (bridging quantum photon production and thermal emission effects).

Within this context, the vast majority of previous works have assumed instantaneous (non-dispersive) time modulations [3, 4, 5, 6], where the material response is temporally local. While mathematically convenient due to its simplicity, this assumption leads to unphysical high-frequency divergences in the emission spectra [4]. In this work, we address this limitation by extending the theoretical formalism to include dispersion (memory effects) in the material response [7]. This analysis not only resolves the issue of the unbounded spectral behavior, but also provides a robust framework for calculating realistic emission spectra in time-modulated materials, offering additional insights that could be instrumental in experimental investigations of intrinsic material properties related to thermal emission effects in time-varying media.

2 Thermal Emission in Time-Varying Media

Traditionally, thermal emission has been based on *fluctuational electrodynamics*, from which the emission spectra can be calculated as the field density correlations by means of the *fluctuation-dissipation theorem* (FDT):

$$\langle \mathbf{j}^*(\mathbf{r};\omega)\cdot\mathbf{j}(\mathbf{r}';\omega')\rangle_{\text{th}} = 4\pi\varepsilon_0\varepsilon''\hbar\omega^2\Theta(\omega,T)\delta_{\mathbf{r}-\mathbf{r}'}\delta_{\omega-\omega'}, \quad (1)$$

where the brackets $\langle\cdots\rangle_{\text{th}}$ denote a thermal ensemble average, $\Theta(\omega,T) = [e^{\hbar\omega/(k_BT)}-1]^{-1}$ is the blackbody's photon distribution, and $\varepsilon(\mathbf{r},\omega) = \varepsilon'(\mathbf{r},\omega) + i\varepsilon''(\mathbf{r},\omega)$ stands for the permittivity of the body (i.e., the linear response function). From this semiclassical framework, and on the basis of the *Green's function formalism*, that relates the electric fields and currents, $\mathbf{E}(\mathbf{r};\omega) \equiv i\omega\mu_0\int d^3\mathbf{r}'\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{j}(\mathbf{r};\omega)$, with $\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$ being the dyadic Green's function characterizing the medium, the thermal emission spectrum (or spectral energy density) can be calculated from the FDT as the field correlations:

$$I(\mathbf{r};\omega) = \langle \mathbf{E}^*(\mathbf{r};\omega)\cdot\mathbf{E}(\mathbf{r};\omega)\rangle_{\text{th}}. \quad (2)$$

2.1 Non-Dispersive Approach

To address thermal emission in time-varying media one necessarily has to take into account the occurrence of both thermal and quantum vacuum electromagnetic (EM) fluctuations, thereby requiring the introduction of a purely quantum formalism [8]. This has been carried out within the framework of macroscopic QED [4], where the dynamical behavior of a time-varying quantum system can be described by means of the Hamiltonian $\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}_{\rm T}$:

$$\hat{\mathscr{H}}_0 = \iint d^3\mathbf{r} d\omega_f \hbar\omega_f \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega_f;t)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega_f;t), \tag{3a}$$

$$\hat{\mathscr{H}}_{\rm T} = -\int d^3\mathbf{r}\hat{\mathbf{P}}(\mathbf{r};t)\cdot\hat{\mathbf{E}}(\mathbf{r};t). \tag{3b}$$

Here, $\hat{\mathscr{H}}_0$ stands for the ground contribution, characterizing the free-evolving polaritons (i.e., the light-matter coupled states) hosted by the macroscopic body without time-modulation, and $\hat{\mathscr{H}}_{\rm T}$ is the interaction term, accounting for the perturbation yielded by the time-modulation described as a polarization field induced by an external electric field, $\hat{\mathbf{P}}(\mathbf{r};t) \equiv \int_0^t d\tau \Delta\chi(\mathbf{r},t,\tau)\hat{\mathbf{E}}(\mathbf{r};\tau)$, with $\Delta\chi(\mathbf{r},t,\tau)$ being the susceptibility, and $\hat{\mathbf{E}}(\mathbf{r};t) = \hat{\mathbf{E}}^{(+)}(\mathbf{r};t) + \hat{\mathbf{E}}^{(-)}(\mathbf{r};t)$ the electric field operator in the time domain, whose positive-frequency component reads as:

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r};t) = \iint d^3\mathbf{r}' d\omega_f \mathbf{G}_{\rm E}(\mathbf{r},\mathbf{r}',\omega_f)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega_f;t), \tag{4}$$

with $\mathbf{G}_{\rm E}(\mathbf{r},\mathbf{r}',\omega_f) = i\sqrt{\hbar/\pi\varepsilon_0}[\omega_f/c]^2\sqrt{\varepsilon''}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega_f)$ being the response function characterizing the background medium, and noticing that $\hat{\mathbf{E}}^{(-)}(\mathbf{r};t) = [\hat{\mathbf{E}}^{(+)}(\mathbf{r};t)]^\dagger$, and that the susceptibility function, $\Delta\chi(\mathbf{r},t,\tau)$, should be small enough so that the time modulation could be regarded as a perturbation to the background structure, at the same time that it does not significantly affect to the quantization. Under this quantum approach, the emission spectrum is:

$$I(\mathbf{r};\omega) = \langle[\hat{\mathbf{E}}^{(+)}(\mathbf{r};\omega)]^\dagger\cdot\hat{\mathbf{E}}^{(+)}(\mathbf{r};\omega)\rangle_{\rm th}, \tag{5}$$

where $\hat{\mathbf{E}}^{(+)}(\mathbf{r};\omega) = \mathscr{L}_\omega[\hat{\mathbf{E}}^{(+)}(\mathbf{r};t)]$ is the Laplace's transform of the electric field operator given in Eq. (4). At the same time, the electric field operator can be recast as:

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r};\omega) = i\omega\mu_0\int d^3\mathbf{r}'\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\hat{\mathbf{j}}(\mathbf{r}';\omega), \tag{6}$$

where $\hat{\mathbf{j}}(\mathbf{r};\omega)$ stands for the current density operator. These latter expressions, relating the EM fields and their sources (i.e., the electric current operators), justify the relationship between the emission spectra and the quantum version of the FDT, clearly showing that the dynamics of the model is ultimately determined by the polaritonic operators. Specifically, working under the Heisenberg picture, the dynamical behavior of the quantum operators characterizing the system directly stems from the Heisenberg equation of motion:

$$i\hbar\partial_t\hat{\mathscr{O}} \equiv \left[\hat{\mathscr{O}},\hat{\mathscr{H}}\right]. \tag{7}$$

Specifically regarding the polaritonic operators, and bearing in mind the canonical equal-time commutation relations, for the free-evolving term of the Hamiltonian given by Eq. (3a) (i.e., that associated to a system without time-modulation), it follows that:

$$i\hbar\partial_t\hat{\mathbf{f}}_0 = \left[\hat{\mathbf{f}}(\mathbf{r},\omega_f;t),\hat{\mathscr{H}}_0\right] = \hbar\omega_f\hat{\mathbf{f}}_0(\mathbf{r},\omega_f;t). \tag{8}$$

Likewise, for the interaction term of the Hamiltonian accounting for the time-modulation, given by Eq. (3b), the corresponding equation generally reads as:

$$\begin{aligned} i\hbar\partial_t\hat{\mathbf{f}}_{\rm T} &= [\hat{\mathbf{f}}(\mathbf{r},\omega_f;t),\hat{\mathscr{H}}_{\rm T}] \\ &= \int d^3\mathbf{r}'\left[\hat{\mathbf{P}}(\mathbf{r}';t)\hat{\mathbf{E}}(\mathbf{r}';t),\hat{\mathbf{f}}(\mathbf{r},\omega_f;t)\right] \\ &= \int d^3\mathbf{r}'\int_0^t d\tau\Delta\chi(\mathbf{r}',t,\tau)\left[\hat{\mathbf{E}}(\mathbf{r}';\tau)\hat{\mathbf{E}}(\mathbf{r}';t),\hat{\mathbf{f}}(\mathbf{r},\omega_f;t)\right]. \end{aligned} \tag{9}$$

In order to evaluate this latter expression, one should notice that the field operators in the last commutator are at different times. A simple manner to circumvent this rather intricate form of time-dependent quantum commutators, consists in assuming a non-dispersive (i.e., temporally-local or instantaneous) time-modulation profile such as:

$$\Delta\chi(\mathbf{r},t,\tau) = \Delta\tilde{\chi}(\mathbf{r},t)\delta[t-\tau]. \tag{10}$$

In doing so, it can be shown that the Heisenberg equation of motion for the time-modulated polaritonic operators is:

$$i\hbar\partial_t\hat{\mathbf{f}}_{\rm T} = -2\int d^3\mathbf{r}'\Delta\tilde{\chi}(\mathbf{r}',t)\mathbf{G}_{\rm E}^*(\mathbf{r}',\mathbf{r},\omega_f)\hat{\mathbf{E}}(\mathbf{r}';t). \tag{11}$$

Thus, the current density operator can be compactly expressed as:

$$\hat{\mathbf{j}}(\mathbf{r};\omega) = 2\omega\left[\sqrt{\pi\hbar\varepsilon_0\varepsilon''}\hat{\mathbf{f}}_0 - i\mathscr{L}_\omega[\Delta\tilde{\chi}(\mathbf{r},t)\hat{\mathbf{E}}(\mathbf{r};t)]\right]. \tag{12}$$

Owing to the time-modulation, the current density operator is implicitly defined in terms of the electric field operator, which is itself generated by the current density operator. Be that as it may, such an equation can be solved by means of an iterative procedure leading to a solution in terms of a series expansion of the current density operators at different orders, $\hat{\mathbf{j}}(\mathbf{r};\omega) = \sum_n\hat{\mathbf{j}}_n(\mathbf{r};\omega)$, whose first elements are:

$$\begin{aligned} &\hat{\mathbf{j}}_0 = \omega\sqrt{4\pi\hbar\varepsilon_0\varepsilon''(\mathbf{r},\omega)}\hat{\mathbf{f}}(\mathbf{r},\omega;t=0); \\ &\hat{\mathbf{j}}_1 \propto \int d^3\mathbf{r}'\int d\omega'\omega'\Delta\tilde{\chi}(\mathbf{r},\omega-\omega')\mathbf{G}(\mathbf{r},\mathbf{r}',\omega')\hat{\mathbf{j}}_0(\mathbf{r}';\omega') \\ &+\int d^3\mathbf{r}'\int d\omega'\omega'\Delta\tilde{\chi}(\mathbf{r},\omega-\omega')\mathbf{G}^*(\mathbf{r},\mathbf{r}',\omega')\hat{\mathbf{j}}_0^\dagger(\mathbf{r}';\omega'); \\ &\hat{\mathbf{j}}_2 \propto \int d^3\mathbf{r}'\int d\omega'\omega'\Delta\tilde{\chi}(\mathbf{r},\omega-\omega')\mathbf{G}(\mathbf{r},\mathbf{r}',\omega')\hat{\mathbf{j}}_1(\mathbf{r}';\omega'). \end{aligned}$$

Accordingly , the current density correlations are given by:

$$\langle\hat{\mathbf{j}}^\dagger(\mathbf{r};\omega)\cdot\hat{\mathbf{j}}(\mathbf{r}';\omega')\rangle_{\rm th} = \sum_{l,m}\langle\hat{\mathbf{j}}_l^\dagger(\mathbf{r};\omega)\cdot\hat{\mathbf{j}}_m(\mathbf{r}';\omega')\rangle_{\rm th}, \tag{13}$$

where the corresponding nth-order can be obtained from a direct evaluation of $\langle\hat{\mathbf{j}}_l^\dagger(\mathbf{r};\omega)\cdot\hat{\mathbf{j}}_m(\mathbf{r}';\omega')\rangle_{\rm th}$, with $l+m=n$. This expression, along with the explicit evaluation of the EM current densities at different orders, can be regarded as a generalization of the FDT for time-varying systems [4].

2.2 Dispersive Approach

Equation (10) represents an unphysical assumption, as it assumes the existence of a memory-less system responding instantaneously to the time-modulation. Despite that, within the quantum domain, it greatly facilitates the mathematical derivation of the Heisenberg equations of motion for the polaritonic operators (and hence the EM current density and field operators as well as the ensuing correlations), and provides with reasonable predictions in the limit of sufficiently low frequencies [4]. However, this non-dispersive approach undertakes significant limitations in fundamental physical issues, which is fostering an increasingly growing interest in addressing dispersion effects in time-varying media [9]. Furthermore, within the context of thermal emission, this assumption leads to a spectral divergence in the high-frequency limit [see solid green curve in Fig. 1(a)]. This realization can be attributed to two main reasons: (1) the Fourier transform of the Dirac delta function in time-domain is a constant function for all frequencies, $\mathscr{F}\{\delta[\tau-t]\}=C$ [see dashed red curve in Fig. 1(a)], and (2) the vacuum energy spectrum becomes comparable and even higher than the blackbody thermal spectrum in a relatively high-frequency limit, $\hbar\omega/2 \gtrsim \hbar\omega\Theta(\omega,T)$ for $\omega>\omega_0$ [see solid black curve in Fig. 1(a)] [8]. Then, given that the time-modulation is a mechanism to dynamically amplify these zero-point quantum vacuum fluctuations, it yields a monotonously increasing contribution at higher frequencies. Therefore, in absence of a dispersive response in the time-modulation that may yield a frequency-tempered function, the emission spectrum shall display a diverging behavior in the high-frequency limit, somehow resembling the *ultraviolet catastrophe* in the Rayleigh-Jeans model [1].

To extend the theoretical model beyond this ideal approach toward a more realistic scenario including dispersive time-modulations [7], we reconsider the above expression for the time-dependent electric field tied to the polarization operator [cf. Eq. (4)], and express it in terms of a Taylor series expansion:

$$\hat{\mathbf{E}}(\mathbf{r};\tau)=\iint d^3\mathbf{r}' d\omega_f \mathbf{G}_{\mathrm{E}}(\mathbf{r},\mathbf{r}',\omega_f)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega_f;t)e^{i\omega_f(\tau-t)}. \quad (14)$$

Here, instead of the Dirac delta function as given in Eq. (10), we model the time-varying susceptibility as its analytical extension regarded as a Gaussian distribution:

$$\Delta\chi(\mathbf{r}',t,\tau)=\Delta\tilde{\chi}(\mathbf{r}',t)\frac{H[t-\tau]}{T_{\mathrm{mem}}\sqrt{\pi}}e^{-(t-\tau)^2/T_{\mathrm{mem}}^2}, \quad (15)$$

where $H[t-\tau]$ stands for the Heaviside step function, and T_{mem} is the temporal width defining the memory-time of the time-modulation [see the schematic representation in Fig. 1(b)]. From these expressions it can be demonstrated that the Heisenberg equation of motion for the time-modulated contribution of the polaritonic operators can be compactly expressed as:

$$i\hbar\partial_t\hat{\mathbf{f}}_{\mathrm{T}}\approx-\Xi_{T_{\mathrm{mem}}}\int d^3\mathbf{r}'\Delta\tilde{\chi}(\mathbf{r}',t)\mathbf{G}_{\mathrm{E}}^{*}(\mathbf{r}',\mathbf{r},\omega_f)\hat{\mathbf{E}}(\mathbf{r}';t), \quad (16)$$

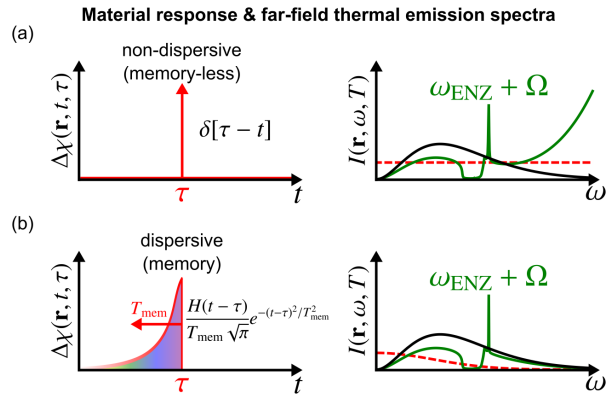

Figure 1. (a) Non-dispersive material responses to the time-modulation yield diverging emission spectra in the high-frequency limit. (b) Dispersive material responses to the time-modulation lead to a cutoff at high-frequencies.

where $\Xi_{T_{\mathrm{mem}}}(t)=e^{-\omega_f^2T_{\mathrm{mem}}^2/4}\mathrm{Erf}[t/T_{\mathrm{mem}}]$, i.e., the so-called Gauss error function. It is worth remarking that this relatively simple expression has been obtained under the condition that $T_{\mathrm{mem}}\omega_f\to 0$, which still restricts the scope of the formalism to systems with very short, though finite, memory times. At any rate, it generalizes previous results, and allows to recovering the non-dispersive (instantaneous) approximation leading to the case of a memory-less system [cf. Eq (11)]. Indeed, in the limit of zero memory, i.e., $T_{\mathrm{mem}}\to 0$, the time-varying susceptibility becomes $\Delta\chi(\mathbf{r}',t,\tau)\to\Delta\tilde{\chi}(\mathbf{r}',t)\delta[t-\tau]/2$ (notice that the Dirac delta function, regarded as a broad Gaussian kernel centered at the time τ, only account for one half of the whole area, leaving aside the other part of the function), so that $i\hbar\partial_t\hat{\mathbf{f}}_{\mathrm{T}}(\mathbf{r},\omega_f;t)\to-\int d^3\tilde{\mathbf{r}}\Delta\tilde{\chi}(\tilde{\mathbf{r}},t)\bar{\mathbf{G}}_{\mathrm{E}}(\tilde{\mathbf{r}},\mathbf{r},\omega_f)\hat{\mathscr{E}}(\tilde{\mathbf{r}};t)$, thus bringing forth the non-dispersive dynamical behavior [4].

This approach can be straightforwardly incorporated into the non-dispersive formalism as a direct extension. Indeed, since the exponential factor in $\Xi_{T_{\mathrm{mem}}}(t)$, i.e., $e^{-\omega_f^2T_{\mathrm{mem}}^2/4}$, is independent of time, it can be taken out of the time integral that yields the expression of time-modulated polaritonic operators. Hence, for both the EM field and current density operators, this contribution appears merely as a multiplicative factor that accompanies all previously derived expressions for temporal modulation (cf. Ref. [4]). Therefore, the dispersive time-modulated contribution to the emission spectra remains the same as in the non-dispersive case, but being weighted by the square of the aforementioned factor, i.e., $e^{-\omega_f^2T_{\mathrm{mem}}^2/2}$, accounting for the correlation of two time-modulated electric field operators.

We illustrate the consequences of introducing the dispersion into the theoretical model by considering the same explanatory example presented in Ref. [4], which consists of a semi-infinite planar slab made of silicon carbide (SiC) ($z<0$), at room temperature (300 K),

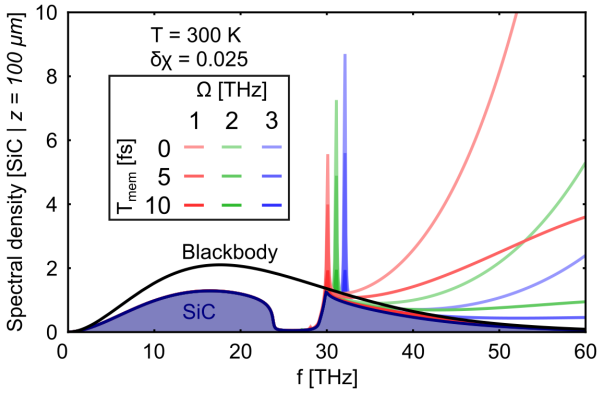

Figure 2. Emission spectra of a semi-infinite planar slab made of SiC at room temperature ($T = 300$ K) with a time-varying susceptibility subjected to a time-harmonic modulation with $\delta\chi = 0.025$ for different values of modulation frequency (Ω) and memory-time (T_{mem}).

in contact with vacuum ($z > 0$). The SiC substrate is optically characterized by the Drude-Lorentz permittivity, $\varepsilon(f) = \varepsilon_\infty(f_{\text{L}}^2 - f^2 - i\gamma f)/(f_{\text{T}}^2 - f^2 - i\gamma f)$, with $\varepsilon_\infty = 6.7$, $f_L = 29.1$ THz, $f_T = 23.8$ THz, and $\gamma = 0.14$ THz, being, respectively, the high-frequency-limit permittivity, the longitudinal and transverse optical phonon frequencies, and the damping factor. In addition, to characterize the modulation we assume that the time-varying susceptibility of the SiC is subjected to a time-harmonic profile:

$$\Delta\tilde{\chi}(\mathbf{r},t) = \varepsilon_0\delta\chi\sin\Omega t. \tag{17}$$

Upon this ground, in Fig. 2 we present the analytical results of the far-field [$z = 100$ μm] emission spectra of this time-modulated dispersive system for different values of the frequency of modulation, Ω, and memory-time, T_{mem}. As can be observed, regardless of the value of modulation frequency Ω, higher dispersion (i.e., longer T_{mem}) leads to sharper cutoff frequencies. Likewise, the intensity of the ENZ-induced emission peak reduces as dispersion effects become dominant.

3 Conclusions

In conclusion, we have extended the theoretical framework of thermal emission in time-varying media to include dispersion effects. This is essentially carried out by introducing a memory-time as an additional parameter to account for the non-instantaneous response to the time-modulation. The key contributions of this work are as follows:

- Identification of a non-physical divergence at high frequencies in the emission spectra of time-modulated materials under a non-dispersive approach;
- Introduction of dispersion into the model, which leads to high-frequency spectral cutoffs;
- More accurate framework to address emission spectra, which enables the computation of the total power radiated by time-modulated materials;
- Theoretical insights for realistically investigating the intrinsic properties of materials in response to external temporal modulations;
- Practical insights for designing experimental setups to observe the effects of thermal emission in time-varying media.

Acknowledgements

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