URSI AP-RASC, Sydney, Australia, 17 – 22 August 2025

Suppression of ultrarelativistic electron acceleration by three-dimensional chorus wave structures

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Abstract

The dynamics of ultrarelativistic electrons in Earth's outer radiation belt are largely determined by resonant interactions with various plasma waves. Whistler-mode chorus emissions drive the local acceleration of these electrons, and theoretical investigations suggest that nonlinear processes can double their energy within seconds. However, amplitude modulations along magnetic field lines can slow down the acceleration process. In this Summary Paper, we explore how perpendicular wave modulations affect these nonlinear resonant interactions. Using modulation scales characteristic of those observed by the Cluster spacecraft, we show that the acceleration process becomes disrupted, leading to highly stochastic electron trajectories on sub-second timescales. These findings suggest that nonlinear effects are negligible in the chorus-driven acceleration of ultrarelativistic electrons, and that the process can be described as pure diffusion in velocity space.

1. Introduction

A central topic in radiation belt research is the interaction between plasma waves and charged particles [1], [2]. When particles meet the resonance condition, scattering can lead to significant changes in energy and pitch angle. Traditionally, this scattering process is described using the quasilinear approximation [3], [4], [5], which assumes small average wave amplitudes and incoherent spectra, resulting in stochastic behavior. However, theoretical investigations and particle simulations have shown that high-amplitude waves can trap particles in their potential [6], [7], rapidly transporting them through velocity space. In the case of whistler-mode chorus waves interacting with electrons, such nonlinear trapping effects could accelerate ultrarelativistic electrons (energy > 2 MeV) by several MeV within seconds [8], [9], [10].

Nevertheless, these predictions of superfast acceleration are based on simplified wave models. Recent studies have revealed that the rising-tone elements in chorus spectrograms are not perfectly coherent but instead display rapid subpacket modulations and phase discontinuities [11], [12]. This decreased coherence limits the efficiency of nonlinear scattering [13], although processes such as successive trapping by adjacent subpackets may still occur, complicating the use of a purely stochastic description on short time scales [14].

In this work, we focus on the effect of perpendicular modulations of the wave field—a factor that has not been considered before. Observations from multi-spacecraft missions have shown that the correlation in the wave field drops rapidly after 50–200 km [15], suggesting that the wave amplitude is modulated on scales corresponding to the gyroradius of MeV electrons. Through test-particle simulations, we demonstrate that these modulations strongly disrupt the nonlinear acceleration process, inducing stochastic behavior in electron trajectories on sub-second timescales. Our findings indicate that the acceleration of ultrarelativistic electrons can be effectively described by a simple diffusion coefficient, eliminating the need for a detailed nonlinear theoretical treatment.

2. Methods

To isolate the effect of perpendicular whistler-mode wave amplitude modulations on electrons with large gyroradii (ultrarelativistic, $E_k > 2$ MeV; see Figure 1a), we employ a simple wave model with no field-aligned modulations. The wave frequency is fixed at $\omega = \Omega_{e0}/4$, where Ω_{e0} is the equatorial electron gyrofrequency at a radial distance of $6\,R_E$ (Earth's radii). Wave propagation is strictly parallel (wave normal angle = 0°), and the wave magnetic amplitude is set to 0.3% of the equatorial dipole field at $6\,R_E$. In the perpendicular direction, we implement Gaussian amplitude modulations of characteristic size w (Figure 1b). This size decreases along the field line to account for flux tube contraction. In addition, the wave field is symmetrized around the source point at the equator, with a 1000 km ramp-up region to prevent nonresonant scattering from amplitude gradients [16].

In the test-particle simulation, electrons are initialized at the equator near $6\,R_E$ and travel northward. The dipole configuration is transformed into a magnetic bottle, and the particles bounce between their mirror points until they either enter the loss cone or reach $t_{max} = 3$ s. Their energies range from 300 keV to 10 MeV in 16 logarithmic steps, while their pitch angles are sampled linearly from 0° to 90° in 16 steps. The gyrophase is uniformly sampled by 360 points. We integrate the equations of motion with a phase-corrected Boris algorithm [17], using adaptive time steps that ensure at least 48 trajectory points per gyroperiod.

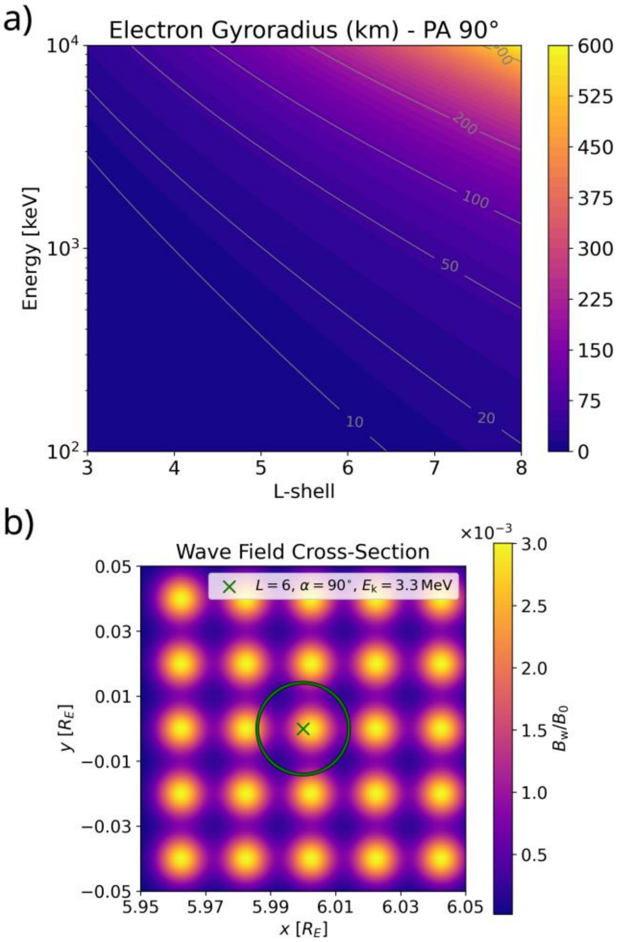

Figure 1. a) Color-coded map of the equatorial electron gyroradius (in km) as a function of L-shell (horizontal axis) and energy (vertical axis) for a pitch angle of 90°. Dipole L-shell values correspond to radial distances in Earth radii at the magnetic equator. b) Equatorial cross section of the simplified wave field model, showing two-dimensional Gaussian modulations in amplitude. The green ring denotes the gyroradius of an ultrarelativistic (3.3 MeV) electron, indicating how its orbit intersects the wave amplitude peaks.

3. Results

In Figure 2, we show the evolution of electron kinetic energy for particles with an initial pitch angle of α=84.38° and an initial energy E_k = 3.34 MeV. Panel a) presents the individual trajectories of particles with different initial gyrophases. After about 0.5 s (less than two bounce periods), some particles begin to get trapped and undergo a very fast ultrarelativistic acceleration (URA) process [9]. As indicated by the histogram on the right, after 3 seconds of evolution, most of the initial population has undergone significant energization. **Fehler! Verweisquelle konnte nicht gefunden werden.**b illustrates the case with a perpendicularly modulated wave field. Here, there is no clear sign of the nonlinear trapping that drives URA. In both the trajectory plot and the histogram, the particles still tend to move toward higher energies, but the spreading is much slower, and the distribution remains single-peaked. This suggests that an approximate description based on inhomogeneous diffusion (and possibly advection) is appropriate.

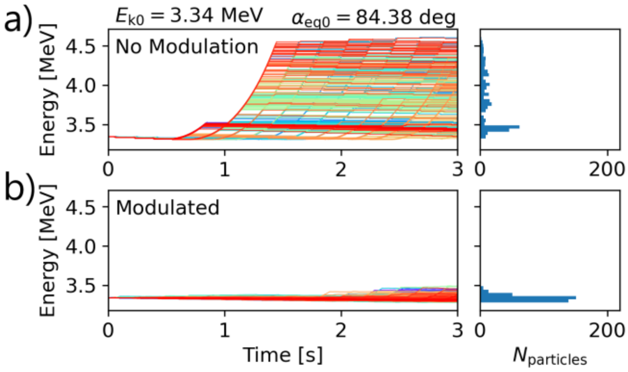

Figure 2. Evolution of electron kinetic energy from test-particle simulation, illustrating the case of ultrarelativistic acceleration for an initial energy of 3.34 MeV and an initial pitch angle of 84.38°. Different line colors correspond to different initial gyrophases. a) Case without wave amplitude modulations. The histogram on the right shows that most particles reach higher energies after 3 s. b) Case with perpendicular wave modulations, where the acceleration is significantly suppressed.

Inspection of individual trajectories reveals that trapping does occur in the presence of perpendicular modulations, but it is short-lived. Figure 3 shows an example of a trapped particle. During the 50 ms interval shown—plotted in terms of parallel velocity and relative phase (gyrophase minus wave phase)—the particle completes fewer than three orbits. The jittery black trajectory reflects the disturbances caused by the wave amplitude modulation. Figure 3 also depicts the boundary (separatrix) of the trapping region [18] at four different time points. Its shape and size change rapidly, with the oscillating particle frequently lying outside the separatrix.

4. Discussion and Conclusion

The results presented here offer an important step toward answering a longstanding question: why do radiation belt models based on quasilinear theory perform well, even in the presence of structured, high-amplitude chorus waves [19]? Building on earlier work that focused on field-aligned modulations [10], [13], we have shown that perpendicular modulations can be even more efficient at disrupting nonlinear acceleration, although this effect predominantly impacts particles with large gyroradii. If perpendicular modulations are ubiquitous, then strong nonlinear energization events may be rare, and diffusion-based approaches may provide an adequate statistical description. This finding opens several promising avenues for future research.

First, a deeper understanding of the three-dimensional structure of high-amplitude waves is necessary. Our study considered only a constant-frequency wave, neglecting the frequency drift typically seen in chorus elements [20]. Moreover, the perfectly regular modulations used in our model are likely unrealistic. Realistic wave fields would include variations in both the size and depth of the modulations, as well as frequency variations that introduce additional phase jumps and potentially lead to even more stochastic behavior. Constructing such a realistic wave field model requires comprehensive observational data, and spatial structures can be reliably characterized only by multiple, closely spaced spacecraft. Our results therefore motivate future multisatellite missions into the outer radiation belt.

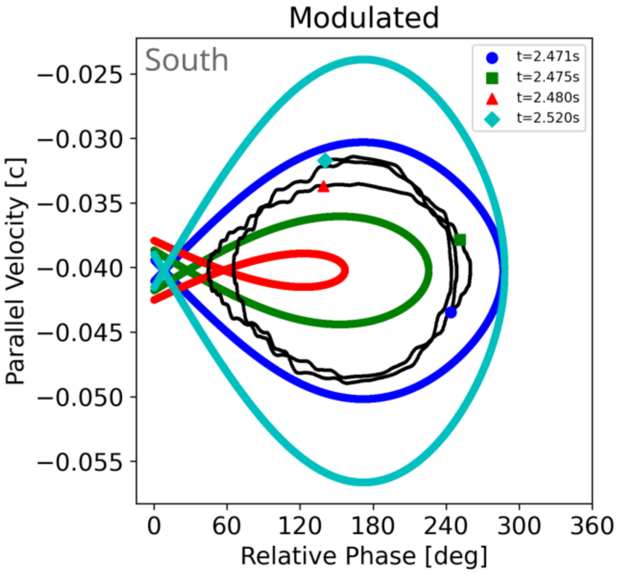

Figure 3. Example of short-lived electron trapping in the modulated wave field. The electron, located in the southern hemisphere, moves in the direction of wave propagation. The black trajectory shows stable trapping over a 50 ms interval. At selected time points, the boundary (separatrix) of the trapping region is indicated by thick colored lines.

Second, the stochastic nature of the particle trajectories in a modulated wave field suggests that diffusion (and possibly advection) coefficients can be derived and compared with quasilinear models. If short-lived trapping persists in more detailed simulations, then the coefficients used in current radiation belt codes may need revision to accurately account for wave-particle interactions under strong wave activity.

Overall, processes that disrupt nonlinear acceleration, such as those described in this paper, must be examined carefully to improve the fidelity of the next generation of radiation belt models.

5. Acknowledgements

MH is supported by the Alexander von Humboldt Postdoctoral Research Fellowship. OS acknowledges funding from the GACR grant 25-18095X, from the European Union's Horizon Europe programme under Grant 101081772--FARBES, and from the Czech MEYS Interexcellence II programme through project LUAUS23152.

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