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# Dynamic Dual-Comb Spectroscopy Beyond Nyquist Limit

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## Abstract

A technique that extends the dynamic range of dual-comb spectroscopy (DCS) beyond the conventional Nyquist limit is proposed. By modeling spectral evolution dynamics and implementing adaptive tracking of the spectral position of the dual-comb signal, our method resolves aliasing artifacts in rapidly varying dual-comb measurement environments. Experimental validation shows accurate spectral recovery over a 5-time aliasing wavelength range, overcoming traditional constraints.

## 1. Introduction

Dual-comb spectroscopy (DCS) has emerged as a transformative technique for high-resolution broadband molecular fingerprinting, leveraging the mutual coherence of two optical frequency combs with slightly different repetition frequencies ($f_r$ and $f_s$) [1]. Compared to conventional Fourier-transform spectroscopy, DCS enables rapid acquisition of absorption spectra with comb-tooth resolution across octave-spanning bandwidths [2, 3]. Recent advancements, including dual-comb lasing schemes [4] and cavity-enhanced photoacoustic implementations [5], have pushed detection sensitivities to sub-picometer levels, and quantum-enhanced detection [5] and programmable combs [6] demonstrating exceptional potential for environmental monitoring and precision metrology . The technique's core advantage stems from its ability to downconvert optical spectral information into the radio-frequency domain through asynchronous optical sampling, enabling high direct digital signal processing with atomic-clock-referenced accuracy[7].

Although dual-comb spectroscopy (DCS) exhibits advantages of high precision and rapid refresh rate, they still face inherent bandwidth constraints where measurable spectral span is limited by the maximum unaliased span $S_{nyquist}$ determined by $f_r$ and $f_s$.

The issue of restricted measurement bandwidth becomes particularly evident when dynamic spectral measurements are required[8]. While increasing repetition rates to GHz[9] levels can extend $f_{nyquist}$, such approaches introduce technical complexity and still didn't solve this problem fundamentally.

To address this limitation, we developed an adaptive dual-comb spectral feature tracking algorithm that can automatically detect where folding occurs and applies dynamic offset compensation based on the folding situation, therefore extends the detection spectral coverage beyond Nyquist constraints.

## 2. Method

We extend traditional Kalman filtering to third-order state-space modeling for enhanced dynamic tracking:

$$x_{k+1} = A_k x_k + n_k$$

$x_{k+1}$ is state vector, defined as:

$$x_{k+1} = \begin{bmatrix} f_k \\ \dot{f}_k \\ \ddot{f}_k \end{bmatrix} = \begin{bmatrix} \text{Instantaneous wavelength} \\ \text{Wavelength velocity} \\ \text{Wavelength acceleration} \end{bmatrix}$$

$A_k$ is state transition matrix, defined as:

$$A = \begin{bmatrix} 1 & \Delta t & 0.5\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

$n_k$ is the noise.

To address spectral folding, when the spectral feature approaches the folding boundaries, we employ a 3rd-order Kalman filter to predict the trend of measurable spectral features.

The Kalman filter uses the spectral features from the previous periods to predict the spectral feature for the current period. The filter determines whether folding has occurred based on the predicted trend and the actual measured spectral feature. The 3rd-order Kalman filter ensures that the rate of frequency change between the predicted value and the previously measured values is continuous. However, when spectral folding occurs, the rate of frequency change becomes discontinuous. As a result, the trend of the predicted value will be opposite to that of the actual measured value. This characteristic allows us to detect the occurrence of folding. If folding is detected, the spectral feature is compensate based on the folding situation so that the real wavelength position of the signal will be restored.

## 3. Experimental setup

In this experiment, a single-cavity dual-comb source is employed to generate two optical pulses trains with different repetition rates ($f_r$=47.1 MHz and $f_s$=47.37 MHz). The two pulse trains underwent amplification and spectral broadening via EDFA, resulting in an 18 nm spectral overlap range that enables detection of spectral signals across the entire 18 nm bandwidth. However, due to the Nyquist limit, the maximum unaliased span $S_{nyquist}$ is limited by the repetition rates and their difference.

The signal pulse is introduced into a fiber Bragg grating (FBG) sensor. When the FBG experiences strain, the central frequency of the reflected spectrum from the FBG shifts accordingly. The reflected signal from the

FBG is combined with the local pulse and fed into a balanced photodetector, resulting in a beat signal.

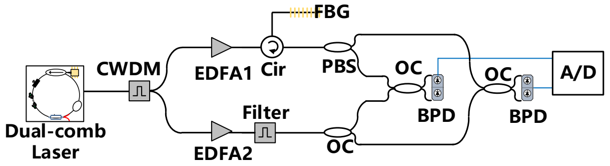


**Figure 1.**Schematic of the experimental setup; EDFA: Erbium-Doped Fiber Amplifier; OC: optical coupler.

By applying dynamic strain to a fiber Bragg grating (FBG), we induced continuous center wavelength shifts of its reflection spectrum from 1541 nm to 1558 nm over a 1.5s period (total variation: 17.2 nm). As shown in Figure 2, optical spectrum analyzer measurements confirmed three characteristic wavelengths: minimum (1541.0 nm), intermediate (1548.7 nm), and maximum (1558.2 nm).

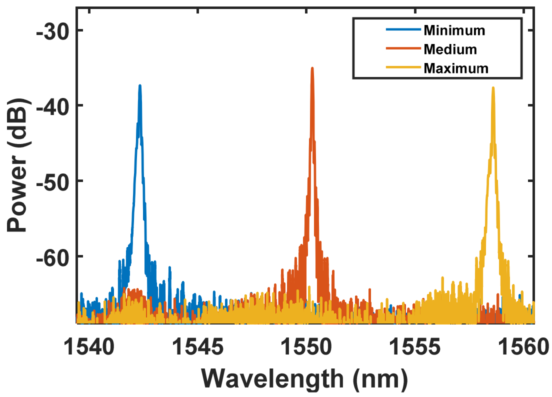


**Figure 2.** Reflection spectrum measured by the optical spectrum analyzer when applying dynamic strain to the FBG.

## 4. Results

Reflection wavelength measured by DCS is shown in figure 3, due to Nyquist constraints, wavelengths reconstructed by conventional dual-comb processing method of the beat signal is limited within one Nyquist span, despite actual shifts spanning 17 nm.

Implementing the Kalman-filter-based adaptive tracking scheme, we achieved accurate recovery of the true spectral variation trajectory (red line). The algorithm address spectral folding events through predict the spectral feature for the current period from the previous periods and applied dynamic offset compensation to unfold aliased components. The compensated results exhibit a maximum deviation of 0.02 nm across the 17 nm range, validating the method's ability to transcend conventional Nyquist limits.

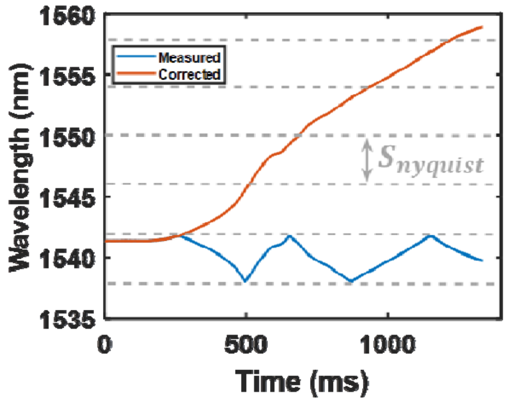


**Figure 3.** Measured spectral features by conventional dual-comb processing (blue squares); Measured spectral features by Kalman-filter-based adaptive compensation (red squares).

## 5. Conclusion

By implementing the Kalman-filter-based adaptive dual-comb spectral feature tracking scheme, we achieve 5 time extended dynamic range beyond the conventional Nyquist limit when performing dynamic dual-comb measurements of spectra with narrow spectral features.

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