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Novel Parameter Extraction of a Surface Impedance for Frequency-Dependent FDTD Analysis

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The finite-difference time-domain (FDTD) method has been widely used as one of the numerical analysis methods for electromagnetic problems. The surface impedance method (SIM) is often used in FDTD analysis including lossy media [1]. Analysis using the SIM allows to use a large mesh size, which can greatly reduce the computational resources. Previously, a novel numerical analysis scheme based on the SIM has been proposed using the first inverse Laplace transform (FILT) and Prony's method [2]. In this scheme, the surface impedance in the frequency domain is converted into the transfer function in z-domain, and the parameters are extracted and incorporated into the FDTD method. However, the proposed scheme has a problem that the time step size used for parameter extraction must be the same as that used in the FDTD method.

In this article, we propose a new parameter extraction approach to solve this problem. First, the transfer function $X(z)$ is obtained using the procedures in [3] as

$$X(z) = \frac{\sum_{j=0}^{N_f-1} b_j z^{-j}}{1+\sum_{i=1}^{N_f} a_i z^{-i}} = \sum_{l=1}^{N_l} \frac{A_l}{1-p_l z^{-1}} + \sum_{k=1}^{N_k} \frac{B_k - C_k z^{-1}}{1-r_k z^{-1}+q_k z^{-2}} \tag{1}$$

where N_f is the total number of poles in the infinite impulse response (IIR) system, N_l and N_k are the numbers of real poles and complex pole pairs, respectively. Next, $X(z)$ is converted to the frequency response $X(\omega)$ as

$$X(\omega) = \sum_{l=1}^{N_l} \frac{\Delta\chi_l}{1+j\omega\tau_l} + \sum_{k=1}^{N_k} \left\{ L_{1,k} \frac{\beta_k}{(j\omega+\alpha_k)^2+{\beta_k}^2} + L_{2,k} \frac{j\omega+\alpha_k}{(j\omega+\alpha_k)^2+{\beta_k}^2} \right\} \tag{2}$$

$$\tau_l = -\frac{\Delta t}{\ln p_l}, \quad \Delta\chi_l = -\frac{A_l}{\ln p_l}, \quad \alpha_k = -\frac{\ln q_k}{2\Delta t}, \quad \beta_k = -\frac{1}{\Delta t}\cos^{-1}\left(\frac{r_k}{2\sqrt{q_k}}\right), \quad L_{1,k} = \frac{B_k r_k - C_k}{\Delta t\sqrt{4q_k - {r_k}^2}}, \quad L_{2,k} = \frac{B_k}{\Delta t}$$

where Δt is the time step size when extracting parameters. Finally, $X(\omega)$ is converted back to a transfer function $X'(z)$ with the time step size $\Delta t'$ that is used in the FDTD method as

$$X'(z) = \sum_{l=1}^{N_l} \frac{A'_l}{1-p'_l z^{-1}} + \sum_{k=1}^{N_k} \frac{B'_k - C'_k z^{-1}}{1-r'_k z^{-1}+q'_k z^{-2}} \tag{3}$$

$$A'_l = \frac{\Delta\chi_l \Delta t'}{\tau_l}, \quad B'_k = L_{2,k}\Delta t', \quad C'_k = L_{2,k}\Delta t' \exp\left(-\alpha_k \Delta t'\right)\cos\left(\beta_k \Delta t'\right) - L_{1,k}\Delta t' \exp\left(-\alpha_k \Delta t'\right)\sin\left(\beta_k \Delta t'\right),$$

$$p'_l = \exp\left(-\frac{\Delta t'}{\tau_l}\right), \quad q'_k = \exp\left(-2\alpha_k \Delta t'\right), \quad r'_k = 2\exp\left(-\alpha_k \Delta t'\right)\cos\left(\beta_k \Delta t'\right)$$

By extracting each parameter from Eq.(3), the surface impedance can be introduced into the FDTD method without any limitations on the time step size.

References

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