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Analysis of Mutual Coupling Using the Asymptotic Friis Formula

Ilkyu Kim(1) and Jeong-Hae Lee(2)
(1) Sejong Cyber University, Seoul, 05000, South Korea
(2) Hongik University, Seoul, 03992, South Korea

Abstract

The accurate prediction of power transmission between two antennas is important for numerous applications, including wireless power transfer and communication link analysis. The classical Friis transmission equation, while reliable in the far-field regime, exhibits reduced accuracy in the near-field regime. In this study, the gain reduction factor including four asymptotic terms have been presented to increase an accuracy in the near-field regime. Utilizing the asymptotic Friis formula with the gain reduction factor for circular apertures, an improved accuracy of the power transmission in a very close near-field regime is obtained. It is demonstrated that the formula is advantageous in terms of predictive accuracy at close distances.

1. Introduction

The successful implementation of electromagnetics and wireless communication applications requires precise estimation of antenna coupling. The classical Friis equation provides an effective solution in the far-field; however, its accuracy deteriorates in the Fresnel region due to deviations from planar wavefronts. To address this limitation, numerous efforts have been made to improve an accuracy in the near-field region. First approach is to use integral form of coupling formula, which is very effective, particularly in close proximity [1], however, relatively complex 3D far-field pattern is required. Another approach is related to the asymptotic Friis formula accounting for gain reduction effect in the near-field regime. Exact gain of circular and rectangular apertures in the near-field regime have been presented, and compared in [2]. Modified Friis formulas with asymptotic correction terms have been presented to improve precision in the Fresnel regime [3]. Another study introduced a gain reduction factor including separation distances normalized with antenna far-field gain [4], which is advantageous in practical use. However, despites its advantages, effective closest distances are limited to the Fresnel regime.

In this paper, a modified Friis formula with asymptotic gain reduction factor including higher order of asymptotic terms is revisited to obtain a more precise prediction in the near-field regime. An optical near-field gain for circular apertures as established in [3] is utilized, and a comparative analysis is conducted with different order of asymptotic terms. The power transmission between two identical circular apertures is obtained, and the effect using different order of asymptotic formulas are evaluated. It is demonstrated that including higher order terms is effective in achieving enhanced precision in the near-field regime.

2. Theory

The power transmission level $|S_{21}|^2$, P_r/P_t can be obtained using Friis power transmission formula,

$$|S_{21}|^2 = \frac{P_r}{P_t} = \frac{\lambda^2}{16\pi^2 R^2} G_t G_r \quad (1).$$

where G_t and G_r denote the far-field gains of the transmitting and receiving antennas, and R is the separation distance. In the near-field regime, the gain reduction effect must be considered in the Friis power transmission formula. The gain reduction factors $\gamma_t(R)$ and $\gamma_r(R)$ provides corrections to the Friis formula,

$$\frac{P_r}{P_t} = \frac{\lambda^2}{16\pi^2 R^2} G_t \gamma_t(R) G_r \gamma_r(R) \quad (2).$$

Exact form of the gain reduction factor for circular aperture [2] can be expressed as

$$\gamma(\Delta) = \left(\frac{\sin \Delta}{\Delta}\right)^2 \quad (3).$$

where Δ is the distance R normalized with $(2D^2)/\lambda$, and D is a diameter of circular aperture. Using a Taylor expansion, the gain reduction factor is approximated using a two-term asymptotic expansion,

$$\gamma_2(\Delta) = 1 - \alpha\Delta^{-2} + \mathrm{O}(\Delta^{-4}) \quad (4).$$

$$\gamma_4(\Delta) = 1 - \alpha\Delta^{-2} + \beta\Delta^{-4} - \delta\Delta^{-6} + \mathrm{O}(\Delta^{-10}) \quad (5).$$

where $\gamma_2(\Delta)$ and $\gamma_4(\Delta)$ are second and fourth order of asymptotic gain reduction factor, respectively, and α, β, δ, and ε are coefficients derived from the optical near-field gain model for circular apertures [3], which are provided in Table I. This fourth order formulation significantly enhances accuracy, particularly in the near-field regime, where prior second order one exhibited some deviations.

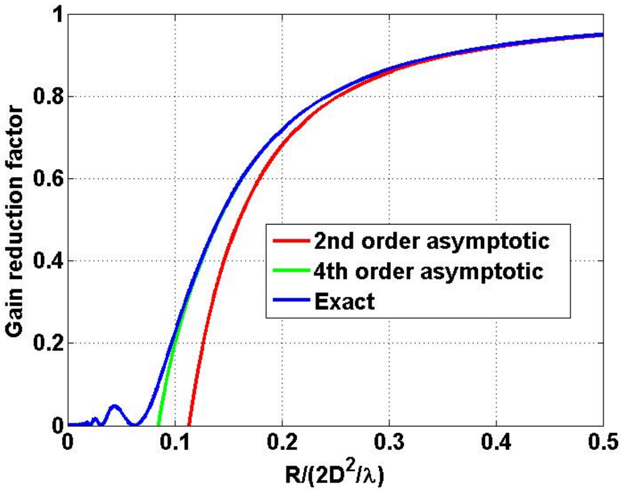

Figure 1. Comparison of second order, fourth order, and exact form of the gain reduction factors for a circular aperture.

Table 2. Coefficients for asymptotic formulas [3].

	α	β	δ
Values	$\frac{1}{3}\left(\frac{\pi}{16}\right)^2$	$\frac{2}{45}\left(\frac{\pi}{16}\right)^4$	$\frac{1}{315}\left(\frac{\pi}{16}\right)^6$

3. Evaluation of Gain Reduction Factors

The second and fourth order of gain reduction factors are calculated using the coefficient presented in Table 1 [3], and compared to the exact form of gain reduction factor. Figure 1 shows a comparison of different gain reduction factors. While second order of gain reduction factor begins to deviate at relatively far distances in the near-field regime, fourth order of gain reduction factor shows an excellent agreement with the exact one within short distances in the near-field regime.

4. Power Transmission

In this session, different Friis formulas presented in (2) are evaluated. For the purpose of evaluation, an antenna gain used in the formulas is presented below,

$$G = \frac{4\pi A_e}{\lambda^2} \qquad (6).$$

where A_e is an effective aperture area of the circular aperture and it is assumed that $A_e = A$(physical aperture area). The gain of the circular aperture with $D = 5\lambda$ is obtained as 23.9 dB. The power transmissions of different asymptotic Friis formulas are calculated using the antenna gain of the circular aperture. Figure 2 shows calculated power transmissions between two identical circular aperture antennas. It is observed that the two asymptotic Friis formula agrees well with the far-field Friis formula in the far-field region. However, as the distance decreases, the graphs of asymptotic formulas are placed below the graph

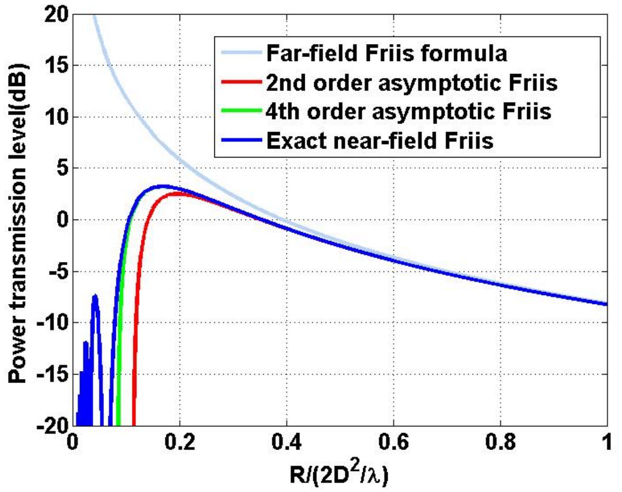

Figure 2. Comparison of power transmission using different orders of asymptotic formula and the exact Friis formula.

Table 2. Nearest distance of the effective range (in terms of $2D^2/\lambda$).

	2nd order	4th order
Distance	0.25	0.17

of the far-field Friis formula, and rapidly decrease at a certain point in the near-field regime. The closest distances of the effective range are presented in Table 2. It is shown that fourth order formula provides 30% closer effective near-field regime, compared to the second order formula.

5. Conclusion

In this paper, we evaluated the accuracy and effective range of different order of asymptotic formulas. It is shown that higher order asymptotic formula provides more accurate results and closer effective range compared to the classical asymptotic formula.

6. Acknowledgement

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