



Particle Scattering Screen Model for Vortex Beam Propagation in Turbid Water

Mingjian Cheng⁽¹⁾, Yuancong Cao⁽¹⁾, and Lixin Guo^{*(1)}

(1) School of Physics, Xidian University, Xi'an, China, 710071, e-mail: mjcheng@xidian.edu.cn; yccao_xd@163.com; lxguo@xidian.edu.cn

Abstract

Underwater optical communication faces significant challenges due to the absorption and scattering effects of water and suspended particles in turbid environments. In this study, we propose a novel and computationally efficient model for simulating particle scattering screens, which integrates both the water absorption properties and the particle size distribution. This model allows for a detailed investigation of vortex beam propagation in turbid water, examining how variations in particle concentration and size influence transmission characteristics. By offering a fast and reliable approach, this model enables effective simulation of vortex beam behavior in complex underwater conditions. The findings provide valuable insights that could significantly enhance underwater optical communication systems and remote sensing technologies, pushing the boundaries of current research in these fields.

1. Introduction

The rapid evolution of communication technologies has led to an increasing demand for robust wireless systems capable of operating in challenging environments, particularly underwater. In this context, underwater optical communication has emerged as a promising alternative to traditional acoustic and radio-frequency methods. It offers distinct advantages, including higher bandwidth, lower latency, and greater resistance to interference [1]. However, optical signal propagation is significantly influenced by scattering and absorption in underwater environments, especially in turbid waters, where suspended particles play a dominant role in signal attenuation.

Early studies in underwater optical communication primarily focused on laser transmission in pure water, identifying the blue-green wavelength range (450–550 nm) as a “transmission window” due to its relatively low attenuation [2]. This discovery prompted further investigations into more complex environments, such as turbid waters with suspended particles, and dynamic conditions involving temperature gradients and fluid flow [3]. These environments introduce additional challenges, such as multiple scattering and dynamic distortions, complicating the prediction of optical signal behavior and necessitating the development of theoretical models.

The advent of structured light, particularly vortex beams, has marked a transformative shift in optical communication. Vortex beams, characterized by helical phase fronts and ring-shaped intensity distributions, enable high-dimensional information encoding through orbital angular momentum (OAM). These beams hold great promise for enhancing underwater communication systems.

However, the propagation dynamics of vortex beams in turbid environments, including the preservation of phase information and OAM states, remain inadequately understood.

To address these challenges, the Generalized Lorenz-Mie Theory (GLMT) [4], an extension of Mie theory for Gaussian and structured beams, was developed to study vortex beam scattering by individual particles. While GLMT provides valuable insights, its application to dense particle suspensions is limited, as it struggles to account for multiple scattering and off-axis effects, thus restricting its usefulness in real-world turbid environments. On the other hand, Monte Carlo methods, commonly used to simulate photon transport in scattering media [5], excel at predicting intensity distributions but often neglect phase characteristics and the evolution of the OAM spectrum in vortex beams. This gap in understanding hampers the optimization of structured light-based communication systems, which require a comprehensive approach to both intensity and phase dynamics.

Despite these advances, the propagation of vortex beams in turbid water remains an open research problem. To fill this gap, this study introduces a novel particle scattering screen model that integrates suspended particle size distribution and water absorption properties. By exploring the attenuation and intensity dynamics of vortex beams in turbid water, this model takes into account varying particle concentrations and sizes. This research provides new insights into the propagation behavior of vortex beams in complex scattering media, offering significant potential for advancing underwater optical communication and remote sensing technologies.

2. Theory model

In a static aqueous medium, laser transmission is primarily affected by the absorption characteristics of water, which exhibit significant wavelength dependence. The complex absorption coefficient of water, $m(\lambda)$, can be expressed as $m(\lambda) = m_1(\lambda) + m_2(\lambda)i$, where $m_1(\lambda)$ represents the real part, associated with scattering, and $m_2(\lambda)$ is the imaginary part, corresponding to absorption. The absorption coefficient at different wavelengths in pure water is related to the complex refractive index through:

$$\alpha = 2\pi m_2(\lambda) / \lambda \quad (1).$$

In turbid water, the refractive and absorption properties of suspended particles must be considered. The scattering behavior of these particles is commonly modeled using

Geometric Optics. By analyzing the refractive indices and sizes of the particles, we can construct particle scattering screens to simulate the scattering properties of the turbid medium. Here, we adopt a step-by-step transmission model, treating the medium as a series of discrete particle scattering screens separated by intervals of Δz . Each screen has dimensions $a \times a$, with randomly distributed circular absorption zones, representing suspended particles. The diameter D_i of each zone corresponds to the size of the simulated particle. The transmission function for a single scattering screen is given by:

$$H_i(x, y) = 1 - \sum_{i=1}^{N_0} P_i(x, y) \quad (2).$$

Here, $P_i(x, y)$ is the absorption function of a single suspended particle, reflecting the random spatial and size distribution of the particles. The total number of particles, N_0 , on each screen corresponds to the number of particles in a volume $\Delta z \times a \times a$. The absorption function satisfies:

$$P_i(x, y) = \begin{cases} A, & \text{if } \left[\left(x - \frac{a}{2} \mathbf{H}_i[0] \right)^2 + \left(y - \frac{a}{2} \mathbf{H}_i[1] \right)^2 \right]^{1/2} \leq \frac{1}{2} D_i \\ 0, & \text{otherwise} \end{cases} \quad (3).$$

where A is the absorption coefficient of the suspended particles, and for dust particles, we assume a fully absorptive model, i.e., $A=1$. The matrix H_i models the random distribution of particles with zero mean and unit variance. The particle size D_i follows a Log-Normal distribution, with the size distribution function given by:

$$p(D_i) = \frac{1}{\sqrt{2\pi}\sigma D_i} \exp\left(-\frac{(\ln D_i - \delta)^2}{2\sigma^2}\right) \quad (4).$$

where σ and δ are the logarithmic standard deviation and mean of the particle size, respectively. The particle size within the medium can be expressed as:

$$D_i = \begin{cases} D_{\min}, & \text{if } D_i < D_{\min} \\ D_{\max}, & \text{if } D_i > D_{\min} \\ \exp(\sigma + \delta \cdot Z_i), & \text{otherwise} \end{cases} \quad (5).$$

where Z_i is a standard normal random variable.

Vortex beam propagates in the turbid environment, undergoing multiple scattering events. Initially, the vortex beam at the source has the intensity distribution given by:

$$U_0(r, \phi, 0) = \left(\frac{r}{w_0}\right)^l \exp\left(-\frac{r^2}{w_0^2}\right) \exp(-il\phi) \quad (6).$$

where r and ϕ are the radial and angular coordinates in cylindrical coordinates, respectively, and w_0 is the beam waist. The OAM mode, l , is carried by the vortex beam.

The vortex beam propagates freely for a distance $\Delta z/2$, where diffraction causes the intensity distribution to evolve. Upon reaching a particle scattering screen, the beam is scattered, altering both its phase and intensity. This process is modeled as:

$$U_1(x, y, \Delta z/2) = \text{IFFT}\left\{\text{FFT}\left[U_0(x, y, 0)H_1(x, y)\right] \times \exp\left(-\frac{ik_r^2 \Delta z}{4k}\right) \exp\left(-\frac{\alpha \cdot \Delta z}{2}\right)\right\} \quad (7).$$

where $U_0(x, y, 0)$ and $U_1(x, y, \Delta z/2)$ are the field distributions before and after the first scattering screen, respectively. The Fourier transform (FFT) and inverse Fourier transform (IFFT) are used to convert between spatial and frequency domains. The wavenumber $k=2\pi/\lambda$ and spatial frequency components are represented by $k_r^2 = k_x^2 + k_y^2$.

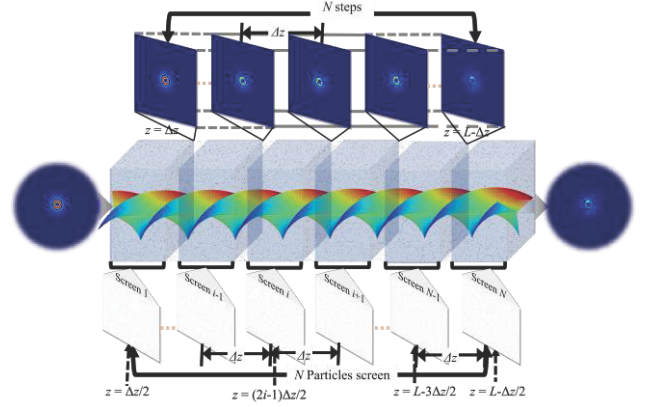


Figure 1. Step-by-step simulation of vortex beam propagation in a turbid underwater environment. The simulation begins with the initial intensity distribution of the vortex beam at $z=0$, followed by free-space propagation over a distance of $\Delta z/2$. A particle scattering screen is then introduced, and the beam undergoes further vacuum propagation of Δz . This process is repeated for s cycles, each with a new particle scattering screen, resulting in a cumulative evolution of the vortex beam's intensity distribution in the scattering medium.

As shown in Figure 1, after each scattering event, the vortex beam propagates further in a vacuum for a distance of Δz , experiencing diffraction that causes the beam to spread. At each step, a new scattering screen is introduced with randomly rearranged particles, simulating the dynamic nature of a turbid medium. Over multiple cycles, the beam's intensity evolves, reflecting the cumulative effects of diffraction and scattering. The total propagation distance is $L=s \times \Delta z$, where s is the number of scattering cycles. At each cycle, the intensity distribution is recorded to analyze the scattering impact on the vortex beam. The light intensity transmittance at the s -th screen is given by:

$$T_s = \frac{\iint U(x, y, z)U^*(x, y, z) dx dy}{\iint U(x, y, 0)U^*(x, y, 0) dx dy} \quad (8).$$

where $U(x, y, z)$ and $U(x, y, 0)$ represent the light intensity transmitted at distance z in turbid water and at the source, respectively. The correlation coefficient between the disturbed and undisturbed light fields is computed:

$$\text{Corr} = \frac{\text{Cov}[U_p(x, y, z), U(x, y, z)]}{\sqrt{\text{var}[U_p(x, y, z)]} \sqrt{\text{var}[U(x, y, z)]}} \quad (9).$$

where $U_p(x, y, z)$ is the light field in pure water, and $U(x, y, z)$ is the light field in turbid water. $\text{Cov}(A, B)$ is the covariance of the two-dimensional light field matrix, and $\text{var}(A)$ represents the variance. The correlation coefficient Corr ranges from $[-1, 1]$, with $\text{Corr}_{\max}=1$, indicating minimal disturbance of the laser beam due to low turbidity.

3. Simulations and Discussions

This section presents a comprehensive analysis of how turbidity distinctly influences the short-range propagation characteristics of vortex beams. Simulations were conducted under the following parameters, unless specified otherwise: particle concentration $N_0=2 \times 10^5 \text{ m}^{-3}$; particle size distribution parameters $\sigma=-1.2$ and $\delta=0.5$; spatial grid resolution of 1024×1024 ; screen dimensions of $6 \text{ cm} \times 6 \text{ cm}$; number of split steps $s=20$; step size $\Delta z=1 \text{ m}$; total propagation distance $L=20 \text{ m}$; beam waist $w_0=0.01 \text{ m}$; wavelength $\lambda=532 \text{ nm}$; and OAM mode $l=3$. To validate the particle scattering screen model, we compared simulation results with experimental data.

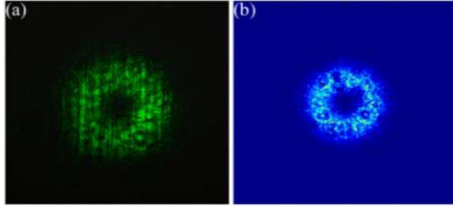


Figure 2. Experimental results of vortex beam propagation through a turbid medium (a) and simulation results with the particle scattering screen (b).

Figure 2 compares the experimental results of vortex beam transmission through a turbid medium (a) with simulation outcomes after transmission through the particle scattering screen (b). Both the experimental and simulated results demonstrate that larger suspended particles obstructing the LG beam generate a ring-shaped diffraction pattern resembling the Airy pattern, thus validating the particle scattering screen model's effectiveness.

Figure 3 illustrates the influence of different OAM modes on the resilience of vortex beam transmission through turbid water. It reveals that vortex beams with higher topological charge numbers ($l \geq 1$), due to their helical phase structure, which forms a ring-shaped energy distribution, are notably more resilient to mode degradation and energy dispersion induced by scattering from suspended particles. The phase singularities responsible for spatial mode separation play a crucial role in reducing correlation interference between the scattered and original light fields, thus slowing down the rate of correlation value decline. In contrast, Gaussian beams ($l=0$) without this structural advantage exhibit optical field statistics that are more susceptible to random scattering, leading to a more rapid decrease in the correlation coefficient.

The concentration of suspended particles in turbid water is directly linked to its turbidity level. We examined the intensity distribution and transmittance of vortex beams propagating through turbid water with particle scattering screens at varying particle concentrations ($1 \times 10^5 \text{ m}^{-3}$, $2 \times 10^5 \text{ m}^{-3}$, and $3 \times 10^5 \text{ m}^{-3}$). At shorter transmission distances, the scattering and absorption effects of

suspended particles on the vortex beam are pronounced. The beam undergoes diffraction, forming a ring-shaped spot, consistent with the results shown in Figure 2. As the transmission distance increases, cumulative scattering and absorption lead to greater energy attenuation. Notably, for the same transmission distance, higher particle concentrations cause more significant distortions in the beam shape. The scattering from different particles overlaps, causing the spot to become increasingly diffuse. After traveling 20 m, the energy transmittance is 28.5%. At particle concentrations of $1 \times 10^5 \text{ m}^{-3}$, $2 \times 10^5 \text{ m}^{-3}$, and $3 \times 10^5 \text{ m}^{-3}$, the energy transmittance at 20 meters is 21.6%, 17.6%, and 13.5%, respectively. These findings highlight that while scattering and absorption by particles significantly affect the beam, the primary cause of energy attenuation is water absorption, which accounts for over 70%.

In addition to particle concentration, particle size distribution significantly varies across different turbid water environments. Particle size is a crucial parameter influencing water turbidity. Using the Log-Normal distribution model to characterize the size of suspended particles, scattering screens were constructed for three distinct particle size conditions, with the following parameters: $\sigma = -1.6$, $\delta = 0.4$; $\sigma = -1.2$, $\delta = 0.5$; and $\sigma = -0.9$, $\delta = 0.6$. At shorter transmission distances, larger particles lead to more pronounced scattering and absorption effects, disrupting the vortex structure to a greater extent. As the transmission distance increases, the fragmentation of the spot becomes more significant. The transmittance of the vortex beam decreases as particle size increases, with larger particles exerting a stronger scattering and absorption effect, leading to lower energy transmittance. After transmitting through four different water environments over 20 m, the energy transmittance values were 28.5%, 22.6%, 17.9%, and 9.4%, respectively.

4. Conclusion

This study introduces an advanced model for underwater suspended particle screens that integrates the absorption attenuation coefficient of water. The model effectively simulates the transmission and attenuation of vortex beams in turbid water, capturing the combined effects of water absorption and suspended particle scattering on laser propagation. The findings reveal that vortex beams with high OAM modes are particularly effective in mitigating mode degradation and energy dispersion induced by particle scattering. The helical phase structure and ring-shaped intensity distribution of these beams enhance spatial mode separation, thereby minimizing interference between the scattered and original light fields, which slows the rate of correlation decline. Although water absorption is the dominant factor contributing to energy attenuation, suspended particles further exacerbate energy loss and disrupt the beam's structure by diffusing its spot. The study shows that higher particle concentrations and larger particle sizes result in more significant attenuation and increased beam distortion. This work underscores the influence of particle concentration and size on transmission characteristics of vortex beams, providing valuable insights for designing laser systems in underwater applications.

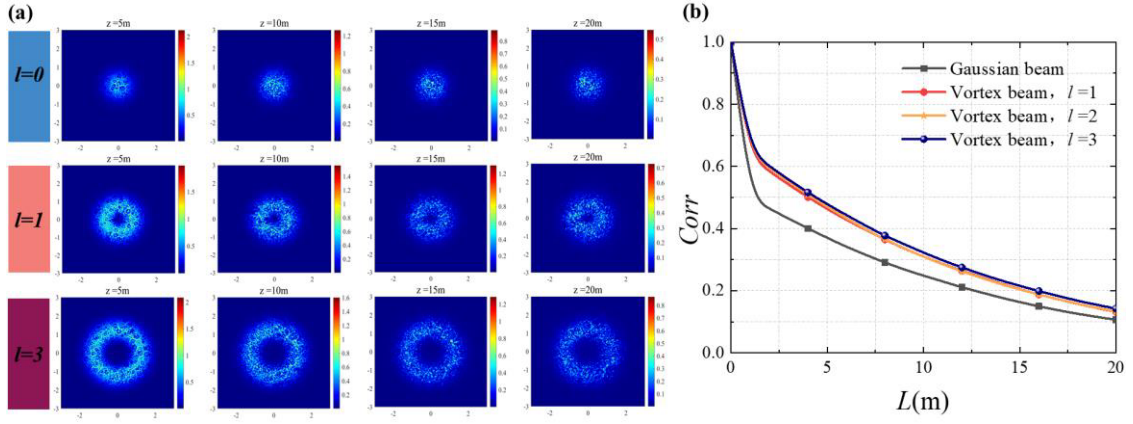


Figure 3. Intensity distributions (a) and correlation coefficients (b) of vortex beams with different OAM modes transmitted over varying distances in turbid water.

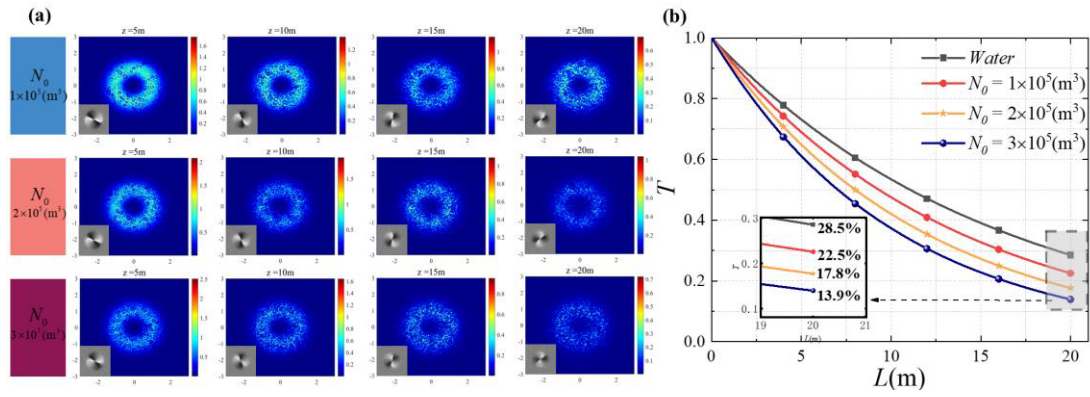


Figure 4. Intensity distributions (a) and transmittance (b) of vortex beam propagation through turbid water at varying particle concentrations.

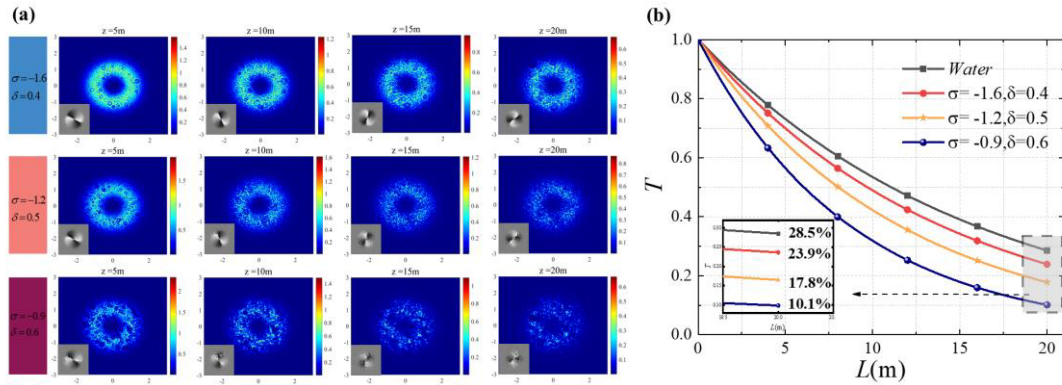


Figure 5. Intensity distributions (a) and transmittance (b) of vortex beam propagation through turbid water with varying particle sizes.

6. Acknowledgements

This work was supported by Shanghai Aerospace Science and Technology Innovation Foundation (SAST-2022-069).

References

- [1] Z. Zeng, S. Fu, H. Zhang, Y. Dong, and J. Cheng, "A survey of underwater optical wireless communications," *IEEE communications surveys & tutorials*, **19**, 1, October 2016, pp. 204-238.
- [2] S. Q. Duntley, "Light in the sea," *JOSA*, **53**, 2, February 1963, pp. 214-233.
- [3] B. Han, K. Köhler, K. Jungnickel, R. Mettin, W. Lauterborn, and A. W. Vogel, "Dynamics of laser-induced bubble pairs," *Journal of Fluid Mechanics*, **771**, May 2015, pp. 706-742.
- [4] G. Gouesbet, B. Maheu, G. Gréhan, "Light scattering from a sphere arbitrarily located in a Gaussian beam, using a Bromwich formulation" *JOSA A*, **5**, 9, March 1988, pp. 1427-1443.
- [5] J. Jönsson, E. Berrocal, "Multi-Scattering software: part I: online accelerated Monte Carlo simulation of light transport through scattering media," *Optics Express*, **28**, 25, 2020, pp. 37612-37638.