URSI AP-RASC, Sydney, Australia, 17-22 August 2025

Formulation of Vector Wave Propagation Over 3D Irregular Terrain

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Propagation of radio waves over irregular terrain is important topic in a number of areas including radar, wireless communications, SAR imaging, etc [1]. When waves propagate over a terrain they can get reflected, deflected, diffracted and attenuated and the amount of signal degradation depends on the slope and composition of the terrain. The extent of terrain involved in these mechanisms depends on the distance between the transmitter and receiver and can even transcend horizon distances. Among the mathematical techniques commonly employed to study propagation are ray-tracing methods, frequency domain and time-domain finite difference methods, frequency domain Fourier transform methods, etc. [2].

Due to the vast computational resources required over terrain, many of the previous studies have focused on the 2D case where the sources and terrain geometry are assumed to be invariant w.r.t. one coordinate. In such a case, the vector problem can be broken down to two scalar problems that can be solved separately. However, a 2D approach completely obscures the aspect of depolarization which couples a vertically polarized field to a horizontal one and vice-versa. Accounting for depolarization could be particularly important in the case of radio imaging system where one component of polarization can become unavailable and adversely affects formation of the image.

In this presentation we discuss the formulation of the full vector problem where the terrain varies in both directions. For simplicity it is assumed to be made of perfectly conducting material and its profile considered twice differentiable. The terrain is described mathematically as $z = \zeta(x, y)$ and its cross-section in the $y = y_1$ plane is shown in Fig. 1. Starting from a covariant form of Maxwell's equations [3], it is shown that the terrain manifests itself as an anisotropic medium over a flat surface with a permittivity and inverse permeability tensors of the form

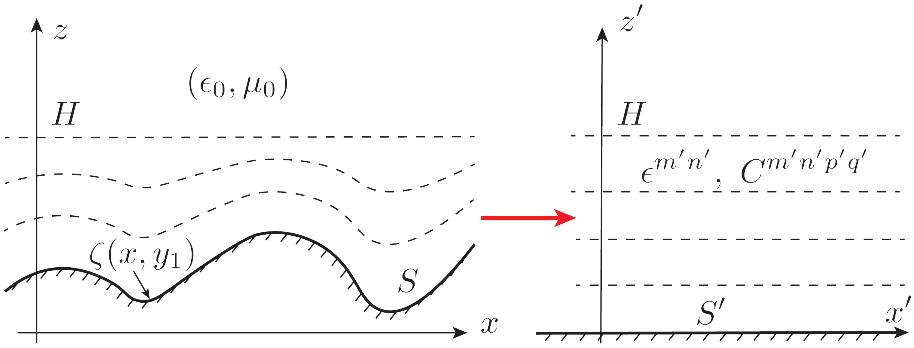

Figure 1. Transformation over an irregular terrain.

$$\epsilon = \mu^{-1} = \begin{bmatrix} h^{-1} & 0 & -g\zeta_x \\ 0 & h^{-1} & -g\zeta_y \\ -g\zeta_x & -g\zeta_y & h\left(g^2\left[\zeta_x^2 + \zeta_y^2\right] + 1\right) \end{bmatrix}; \quad g = \frac{H - z'}{H}, \qquad h = \frac{H}{H - \zeta(x, y)},$$

with the Helmholtz equation getting transformed accordingly.

1. M. W. Long, *Radar Reflectivity of Land and Sea*, 3rd Ed, Artech House, Dedham: MA, 2001.
2. R. Janaswamy, *Radiowave Propagation and Smart Antennas for Wireless Communications*, Kluwer Academic Publishers, MA, 2000.
3. R. Janaswamy, *Engineering Electrodynamics: A Collection of Theorems, Principles and Field Representations*, Institute of Physics, Bristol, UK: 2009.