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Characteristic Modes of Chiral Objects

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Characteristic mode (CM) analysis can be formulated using the integral operators of the corresponding scattering problem. As a step forward in addressing more complex media, we introduce a novel integral-operator-based CM formulation for lossless isotropic chiral objects. Chiral (Pasteur) media is reciprocal and has the constitutive relations

$$\mathbf{D} = \varepsilon\,\mathbf{E} + i\kappa\sqrt{\mu_0\varepsilon_0}\,\mathbf{H}, \qquad \mathbf{B} = \mu\,\mathbf{H} - i\kappa\sqrt{\mu_0\varepsilon_0}\,\mathbf{E}, \tag{1}$$

where the permittivity ε, permeability μ, and chirality parameter κ are real in the lossless case. For simplicity we assume that $|\kappa| < 1$. Fields in chiral media, and more general bi-isotropic media, can most easily be solved using the wavefield decomposition [1] that leads to a sum of two wavefields (denoted with subscripts + and −) in equivalent isotropic media with the same wave impedance $\eta = \sqrt{\mu/\varepsilon}$ but different wavenumbers $k_\pm = \omega\sqrt{\mu\varepsilon}(1 \pm \kappa)$.

Working out the details of the scattering formulation as in [2] and selecting the weight operator following [3], the generalized eigenvalue equation of the new CM formulation is

$$\mathcal{L}\begin{bmatrix}\mathbf{J}_n\\ \mathbf{M}_n\end{bmatrix} = (1 - i\lambda_n)\,\mathcal{M}\begin{bmatrix}\mathbf{J}_n\\ \mathbf{M}_n\end{bmatrix}, \tag{2}$$

where the surface-integral operators $\mathcal{L}$ and $\mathcal{M}$ are

$$\mathcal{L} = \begin{bmatrix}\eta_0\mathcal{T} + \frac{\eta}{2}(\mathcal{T}_+ + \mathcal{T}_-) + \frac{i\eta}{2}(\mathcal{K}_+ - \mathcal{K}_-) & -\mathcal{K} - \frac{1}{2}(\mathcal{K}_+ + \mathcal{K}_-) + \frac{i}{2}(\mathcal{T}_+ - \mathcal{T}_-)\\ \mathcal{K} + \frac{1}{2}(\mathcal{K}_+ + \mathcal{K}_-) - \frac{i}{2}(\mathcal{T}_+ - \mathcal{T}_-) & \frac{1}{\eta_0}\mathcal{T} + \frac{1}{2\eta}(\mathcal{T}_+ + \mathcal{T}_-) + \frac{i}{2\eta}(\mathcal{K}_+ - \mathcal{K}_-)\end{bmatrix}, \quad \mathcal{M} = \begin{bmatrix}\eta_0\,\mathrm{Re}\,\mathcal{T} & -i\,\mathrm{Im}\,\mathcal{K}\\ i\,\mathrm{Im}\,\mathcal{K} & \frac{1}{\eta}\,\mathrm{Re}\,\mathcal{T}\end{bmatrix}, \tag{3}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance of free space. Operators $\mathcal{T}$ and $\mathcal{K}$ are the usual tangential surface integral operators in the exterior domain (free space) with wavenumber $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, while $\mathcal{T}_\pm$ and $\mathcal{K}_\pm$ are the same operators for the two wavefields in the interior domain with wavenumbers $k_\pm$. If $\kappa \to 0$, the formulation simplifies to the PMCHWT formulation [3] for ordinary homogeneous materials.

Assuming that the media is lossless ensures that the characteristic eigenvalues λ_n are real and the characteristic eigencurrents $\mathbf{J}_n, \mathbf{M}_n$ radiate power-orthogonal far fields. Changing the sign of the chirality parameter κ corresponds to taking the mirror image of the material [1]. This change to opposite handedness can be seen in the characteristic eigencurrents $\mathbf{J}_n, \mathbf{M}_n$, but the characteristic eigenvalues λ_n remain the same if κ is replaced with $-\kappa$.

References

[1] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic waves in chiral and bi-isotropic media*, Artech House, 1994

[2] D. Wang, E. K. N. Yung, R. Chen, and P. Y. Lau, "Scattering characteristics of general bi-isotropic objects using surface integral equations," *Radio Science*, **41**, 2, April 2006, RS2008, doi:10.1029/2005RS003315.

[3] P. Ylä-Oijala, "Generalized Theory of Characteristic Modes," *IEEE Transactions on Antennas and Propagation*, **67**, 6, June 2019, pp. 3915–3923, doi: 10.1109/TAP.2019.2905794.