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Three-dimensional Quantitative Imaging by Incorporating TDOA and CSI schemes for Microwave Breast Cancer Ablation Treatment

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Abstract

This paper presents a three-dimensional quantitative imaging method for microwave breast cancer ablation monitoring. To retain a complex permittivity profile during ablation process, this study integrates the time difference of arrival (TDOA) based ablation zone boundary estimation and contrast source inversion (CSI) based inverse scattering schemes. The TDOA based ablation zone prior mitigates the ill-posed problem in the post CSI based inverse scattering analysis. Additionally, to expand the 3D problem, we introduce the cross-sectional imaging scheme, where the 3D data are converted to the 2D data by the transfer function model. The 3D finite difference time domain (FDTD) based numerical tests using MRI-derived breast phantoms show that our proposed scheme successfully reconstructs both permittivity changes and ablation zone with low computational complexity.

1 Introduction

Microwave ablation (MWA) is an emerging and highly promising technique for minimally invasive cancer treatment. Extensive studies have demonstrated its efficacy and safety in treating liver, kidney, and breast cancers [1]. Studies have demonstrated its efficacy and safety for liver, kidney, and breast cancers. In breast cancer ablation, MWA offers significant advantages by minimizing both physical trauma and psychological distress, as it eliminates the need for extensive tissue removal. However, achieving precise and safe tumor ablation while preserving surrounding healthy tissue necessitates the integration of appropriate imaging modalities. Magnetic resonance imaging (MRI) and ultrasound have been investigated for this purpose. MRI provides superior spatial resolution, allowing for detailed visualization of tumor margins. However, its clinical utility is limited by insufficient temporal resolution, high operational costs, and bulky equipment requirements. While MRI provides high spatial resolution, its temporal resolution is often insufficient, and the equipment is costly and large. In contrast, ultrasound imaging is more compact and cost-effective, but its efficacy is compromised by contrast degradation, primarily due to microbubble formation resulting from tissue hydration during the ablation process.

Microwave-based monitoring of the ablation zone offers a simple and cost-effective approach by utilizing only the ablation probe as a transmitting source and surrounding antennas as receivers. Previous studies have shown that tissue dehydration during ablation leads to a significant reduction in permittivity and conductivity, approximately 40 %, which directly impacts microwave propagation characteristics [1]. Consequently, imaging changes in complex permittivity enables a quantitative evaluation of the ablation zone. As a promising solution for this, a tomography (inverse scattering analysis) scheme has been proposed for quantitative imaging of complex permittivity profile. However, this approach faces significant challenges, including high computational costs and inaccuracy due to solving a non-linear and ill-posed inverse problem. As an alternative approach, a time difference of arrival (TDOA)-based ablation zone estimation method has been introduced [2]. This method enables real-time 3D imaging through efficient and simple signal processing. However, the TDOA method cannot provide a quantitative value in complex permittivity change caused by ablation.

To overcome these limitations, we introduce a complex permittivity imaging method that combines the TDOA based ablation zone estimation and contrast source inversion (CSI) tomography [3] scheme. In this approach, a prior knowledge obtained by the TDOA method is exploited to determine the change of complex permittivity, by minimizing the cost function of CSI [4]. Since the literature in [4] assumes the two-dimensional (2D) model, this study extends the method to three-dimensional (3D) scenarios, by extracting the cross-sectional image placed on the observation plane, where the 3D data are converted to the 2D data via the linear transfer function model. The effectiveness of the proposed method is validated through the 3D FDTD based numerical tests using the MRI derived realistic breast phantoms.

2 Method

2.1 Observation Model

The observation model is illustrated in Fig. 1. The transmitting source is positioned near the cancerous zone inside the breast at $\boldsymbol{r}_{\mathrm{T}}$, while receiving elements are placed around the breast at $\boldsymbol{r}_{\mathrm{R}}$. The total electric fields pre and during ablation are denoted as $e_{\mathrm{T}}^{\mathrm{pre}}(t;\boldsymbol{r}_{\mathrm{T}},\boldsymbol{r}_{\mathrm{R}})$ and $e_{\mathrm{T}}^{\mathrm{dur}}(t;\boldsymbol{r}_{\mathrm{T}},\boldsymbol{r}_{\mathrm{R}})$, respectively. The complex permittivity profiles of pre- and during ablation are expressed as $\varepsilon_{\mathrm{pre}}(\omega,\boldsymbol{r})$ and $\varepsilon_{\mathrm{dur}}(\omega,\boldsymbol{r})$.

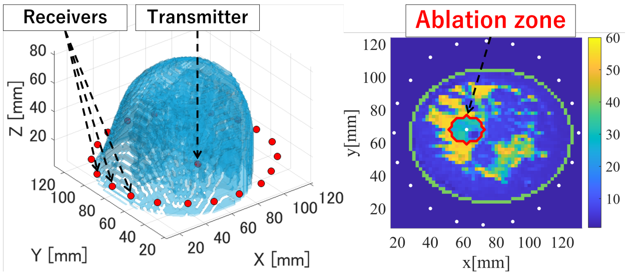

Figure 1. Observation model (Left: 3D breast phantom overview and the arrangements of a single transmitter and multiple receivers, Right: Cross-sectional analysis for the spatial profile of the real part of complex permittivity at 1.04 GHz)

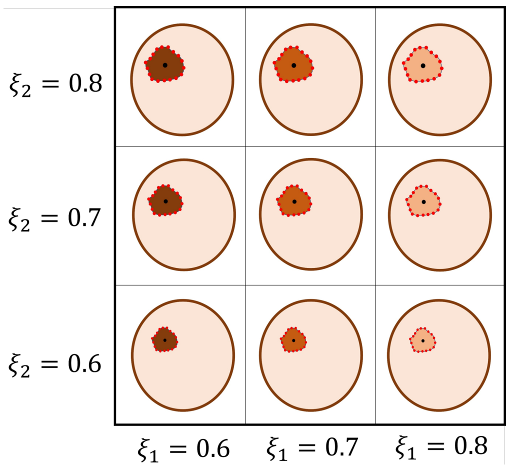

Figure 2. Schematic illustration to determine the parameters ξ_1 and ξ_2 in the proposed method. Red dots show the ablation boundaries.

2.2 Ablation Boundary Estimation Based on TDOA

The proposed method incorporates the ablation boundary estimation by the TDOA method, the methodology of which is briefly described as follows. In this method, the distance $R(\boldsymbol{r}_{\rm R};\xi)$ from the transmitting source $\boldsymbol{r}_{\rm T}$ to the ablation boundary point $\boldsymbol{r}_{\rm B}$ at each receiving position $\boldsymbol{r}_{\rm R}$ is calculated as:

$$R(\boldsymbol{r}_{\rm R};\xi) = \bar{v}_{\rm pre}\frac{\Delta\tau(\boldsymbol{r}_{\rm R})}{1-\sqrt{\xi}}, \quad (1)$$

where $\xi \equiv \bar{\varepsilon}_{\rm dur}/\bar{\varepsilon}_{\rm pre}$. The zone enclosed by the transmitting source $\boldsymbol{r}_{\rm T}$ and the estimated ablation boundary $\boldsymbol{r}_{\rm B}$ is defined as $\Omega_{\rm ab}(\xi)$.

3 TDOA prior CSI

To retain a quantitative change in the ablation zone, our proposed method [4] simultaneously estimates both the ablation zone and its complex permittivity by optimizing the cost function of the CSI method as follows. Notably, the CSI method reconstructs the complex permittivity within the region of interest (ROI) $\Omega_{\rm D}$ by solving the domain integral equation (DIE) as :

$$E^{\rm S}_{\rm dur}(\omega;\boldsymbol{r}_{\rm T},\boldsymbol{r}_{\rm R}) = k_{\rm B}^2\int_{\Omega_{\rm D}} G_{\rm B}(\omega;\boldsymbol{r},\boldsymbol{r}_{\rm R})\mathscr{W}(\omega;\boldsymbol{r}_{\rm T},\boldsymbol{r},\boldsymbol{\xi})d\boldsymbol{r} \quad (2)$$

where $k_{\rm B}$ and $G^{\rm B}(\omega;\boldsymbol{r},\boldsymbol{r}_{\rm R})$ are the wave number and Green's function of the background media, respectively. $\mathscr{W}(\omega;\boldsymbol{r}_{\rm T},\boldsymbol{r},\boldsymbol{\xi}) \equiv \chi(\omega;\boldsymbol{r})E^{\rm T}(\omega;\boldsymbol{r}_{\rm T},\boldsymbol{r})$ is defined, where the contrast function forms $\chi(\boldsymbol{r};\boldsymbol{\xi}) \equiv (\varepsilon_{\rm dur}(\omega;\boldsymbol{r},\boldsymbol{\xi}) - \varepsilon_{\rm B}(\boldsymbol{r}))/\varepsilon_B(\boldsymbol{r})$, and $\varepsilon_{\rm B}(\boldsymbol{r})$ express the backgound complex permittivities at position $\boldsymbol{r}$. Here, the complex permittivity profile during ablation state, $\varepsilon_{\rm dur}(\omega,\boldsymbol{r},\boldsymbol{\xi})$, is assumed as:

$$\varepsilon_{\rm dur}(\omega,\boldsymbol{r},\boldsymbol{\xi}) = \begin{cases} \xi_1\varepsilon_{\rm pre}(\omega,\boldsymbol{r}) & (\boldsymbol{r}\in\Omega_{\rm ab}(\xi_2)) \\ \varepsilon_{\rm pre}(\omega,\boldsymbol{r}) & (\boldsymbol{r}\in\overline{\Omega}_{\rm ab}(\xi_2)\cap\Omega_{\rm D}), \end{cases} \quad (3)$$

where $\boldsymbol{\xi} \equiv (\xi_1,\xi_2)$ holds. Then, $\boldsymbol{\xi}$ is optimized as:

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} F(\mathscr{W},\boldsymbol{\xi}). \quad (4)$$

Here, $F(\mathscr{W},\boldsymbol{\xi})$ denotes the minimized cost function in the CSI, where the contrast function χ is fixed in the minimization. This approach allows independent estimation of the ablation zone and complex permittivity changing by optimizing ξ_1 and ξ_2 separately.

Furthermore, to achieve 3D cross-sectional analysis, the 3D data are converted to the 2D data using the following transfer function model. Using the incident electric fields in 2D and 3D, $E_{\rm I}^{\rm 2D}(\omega)$ and $E_{\rm I}^{\rm 3D}(\omega)$, as well as the total electric fields before ablation, $E_{\rm T}^{\rm 2D}(\omega)$ and $E_{\rm T}^{\rm 3D}(\omega)$, the transfer functions $H_{\rm I}(\omega)$ and $H_{\rm T}(\omega)$ are defined as

$$H_{\rm I}(\omega) \equiv \frac{E_{\rm I}^{\rm 2D}(\omega)}{E_{\rm I}^{\rm 3D}(\omega)}, \quad H_{\rm T}(\omega) \equiv \frac{E_{\rm T}^{\rm 2D}(\omega)}{E_{\rm T}^{\rm 3D}(\omega)}. \quad (5)$$

Then, the scattered electric field during ablation, the scattered data in 3D model is converted to that in the 2D model as:

$$\hat{E}_{\rm sca}^{\rm 3D}(\omega) \approx H_{\rm T}(\omega)\hat{E}_{\rm T}^{\rm 3D}(\omega) - H_{\rm I}(\omega)E_{\rm I}^{\rm 3D}(\omega). \quad (6)$$

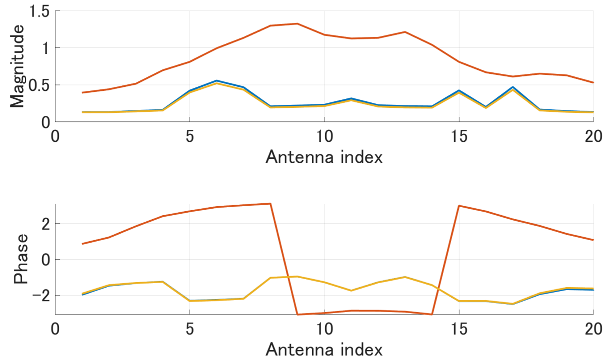

Figure 3. Data conversion results of the scattered electric field from 3D to 2D data. Blue: 2D, Red: 3D (wo/ conversion), Yellow: 3D (w/ conversion).

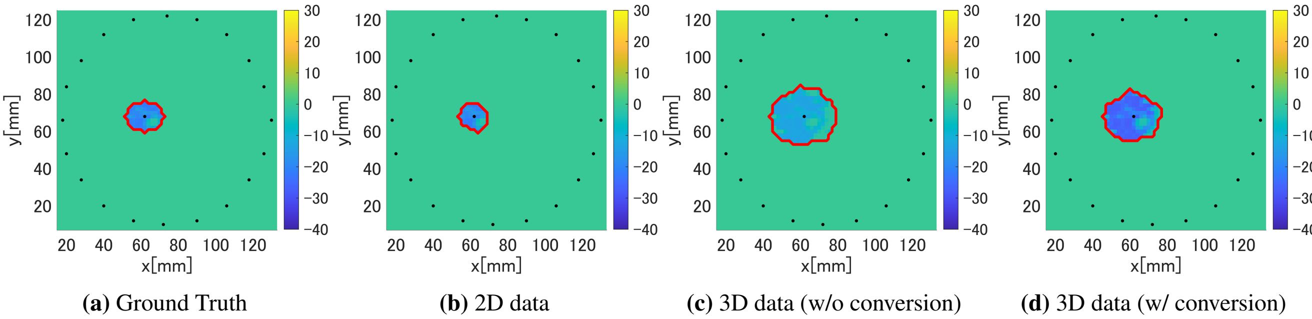

Figure 4. Reconstruction differential profiles between pre- and during ablation states at 1.04 GHz.

4 Results

This study utilizes an MRI-derived Class-3 "heterogeneously dense" phantom (ID: 062204) from the online database [5]. Numerical evaluations are performed by 2D and 3D FDTD simulations, in noise-free situation. The breast model, based on MRI imaging, accurately represents tissue heterogeneity, with dielectric dispersion modeled using a single-pole Debye model. In the simulation setup, the microwave ablation probe serves as the transmitting element and is positioned near the tumor, while 20 receiving elements are strategically placed at the external area of the breast. The transmitted signal has a center frequency of 2.45 GHz and a bandwidth of 1.90 GHz. The ablation zone is approximated as an elliptical region centered at the tumor location, where a uniform 40 % reduction in complex permittivity ($\xi = 0.6$) is assumed. In the proposed method, both parameters ξ_1 and ξ_2 vary from 0.40 to 0.85 in increments of 0.05, generating a total of 81 possible combinations for evaluation.

Figure 3 presents the comparison between the 2D and 3D scattered electric fields in terms of the magnitude and phase, with and without using the data conversion using Eq. 5, at each element position. The results confirm that Eq. 5 enables accurate conversion of 3D data into 2D data. Additionally, Figure 4 illustrates the estimated complex permittivity changes obtained through different methods. While in the case using 2D data, our method retains sufficient accuracy, particular in the ablation zone estimation, in the case using the 3D data, it tends to overestimate the ablation region compared to its 2D counterpart. This is caused by the different propagation and scattering effects between the 2D and 3D models. However, despite this discrepancy, the overall estimation accuracy remains largely consistent.

To quantitatively evaluate the estimation accuracy of the proposed method, we introduce the root mean square error (RMSE) as a metric for evaluating the estimated complex permittivity distribution. Additionally, the estimation error for the ablation zone is defined as

$$\mathrm{Err}_{\Omega_{\mathrm{ab}}} = \frac{\int_{\boldsymbol{r}\in\Omega_{\mathrm{D}}} |\eta^{\mathrm{true}}(\boldsymbol{r}) - \eta^{\mathrm{est}}(\boldsymbol{r})|\, d\boldsymbol{r}}{\int_{\boldsymbol{r}\in\Omega_{\mathrm{D}}} |\eta^{\mathrm{true}}(\boldsymbol{r})|\, d\boldsymbol{r}}. \quad (7)$$

where $\Omega_{\mathrm{ab}}^{\mathrm{true}}$ and $\hat{\Omega}_{\mathrm{ab}}$ denote the ground truth and reconstruction ablation zones, respectively. Here, $\eta^{\mathrm{true}}(\boldsymbol{r})$ and $\eta^{\mathrm{est}}(\boldsymbol{r})$ are defined as

$$\eta^{\mathrm{true}}(\boldsymbol{r}) \equiv \begin{cases} 1 & (\boldsymbol{r} \in \Omega_{\mathrm{ab}}^{\mathrm{true}}) \\ 0 & (\text{otherwise}) \end{cases}, \quad \eta^{\mathrm{est}}(\boldsymbol{r}) \equiv \begin{cases} 1 & (\boldsymbol{r} \in \hat{\Omega}_{\mathrm{ab}}) \\ 0 & (\text{otherwise}) \end{cases}.$$

Table 1. RMSE of real and imaginary part in complex permittivity and the estimation error of the ablation zone.

	RMSE $\Re[\varepsilon]$	RMSE $\Im[\varepsilon]$	$\mathrm{Err}_{\Omega_{\mathrm{ab}}}$
2D	2.18	0.71	0.13
3D (wo/ conversion)	3.27	1.05	0.95
3D (w/ conversion)	5.16	1.70	0.70

Table 1 presents the RMSE and ROI for each dataset. While the converted 3D data exhibits some degradation in RMSE accuracy, it remains effective in estimating the ablation region Potential sources of estimation error include inaccuracies in the transfer function during dimensional conversion and variations in the convergence behavior of the cost function due to the inherent ill-posed nature of the problem.

5 Conclusion

This study proposed a complex permittivity imaging method for microwave ablation therapy of breast cancer, integrating ablation zone estimation via the TDOA method with CSI cost function evaluation. Performance evaluation through 2D and 3D numerical analysis confirmed its effectiveness. Future work will focus on enhancing the accuracy of dimensional conversion using nonlinear transformations.

Acknowledgment

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