URSI AP-RASC, Sydney, Australia, 17 – 22 August 2025

Determining Uniform Linear Array Mutual Coupling Terms Through Multi-Exponential Analysis

Jacki Gilmore
Department of Electrical and Electronic Engineering, Stellenbosch University, South Africa

Abstract

In this work, multi-exponential analysis based on Prony's method is used to solve the mutual coupling terms in uniform linear antenna arrays from active reflection coefficients. The method can be implemented on finite or infinite arrays. Excellent agreement is shown for results of two dipole arrays obtained using this method compared with direct simulation of the S-paramaters using Altair FEKO.

1 Introduction

For many applications using antenna arrays, including multiple input-multiple output (MIMO) systems, direction-of-arrival (DOA) estimation, remote sensing, and radio astronomy, it is critical to have accurate information about the mutual coupling between array elements. However, measuring the mutual-coupling terms in physical systems is a laborious process that is susceptible to errors.

This paper details the first stage of a larger project. The final project aims to derive the mutual coupling terms of arrays from embedded element pattern measurements using multi-exponential analysis based on Prony's method as the mathematical framework [1–3]. This method will be an alternative to the methods proposed in [4] and [5].

As the first part of the larger project described above, this paper formulates the active reflection coefficient of uniform linear arrays (ULA) sampled at various progressive phase shifts as a Prony problem, enabling the S-parameters to be extracted via a Vandermonde structured system.

2 Procedure

For an N-element uniform linear array, the active reflection coefficient of element n for a set of beamformer weights $\mathbf{w_f}$ is [6]

$$\Gamma_n = \frac{1}{w^*_{f,n}} \sum_{p=1}^{N} S_{np} w^*_{f,p}. \tag{1}$$

Now, the beamformer weights are chosen to have unity magnitude and progressive phase α

$$w^*_{f,n} = A_n e^{-j\alpha_n} = e^{-jn\alpha}, \tag{2}$$

$$w^*_{f,p} = A_p e^{-j\alpha_p} = e^{-jp\alpha}, \tag{3}$$

which results in the active reflection coefficient for a given progressive phase α

$$\Gamma_n(\alpha) = \sum_{p=1}^{N} S_{np} e^{j\alpha(n-p)}. \tag{4}$$

To solve all the S_{np} terms, N regularly spaced values of α must be used. The values of α are in the range

$$-\pi \leq \alpha \leq \pi. \tag{5}$$

For this range, the sample values of α can be written as a function of the sample number s

$$\alpha_s = s\Delta - \pi, \;\; s = 0...N-1 \tag{6}$$

$$\Delta = \frac{2\pi}{N}. \tag{7}$$

Now, a Prony model can be derived for the problem:

$$\Gamma_n(\alpha_s) = \sum_{p=1}^{N} S_{np} e^{j(n-p)(s\Delta-\pi)} \tag{8}$$

$$= \sum_{p=1}^{N} S_{np} e^{j(p-n)\pi} e^{j(n-p)\Delta s} \tag{9}$$

$$\Gamma_{ns} = \sum_{p=1}^{N} \beta_{np} \Phi^s_{np}, \tag{10}$$

with the base terms given by

$$\Phi_{np} = e^{j(n-p)\Delta}, \tag{11}$$

and the coefficients given by

$$\beta_{np} = S_{np} e^{j(p-n)\pi}. \tag{12}$$

Note that as the input to the problem, the base terms are already known. The coefficients can now be solved using the Vandermonde structured system

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \Phi_{n1} & \Phi_{n3} & \cdots & \Phi_{nN} \\ \vdots & \vdots & & \vdots \\ \Phi^{N-1}_{n1} & \Phi^{N-1}_{n2} & \cdots & \Phi^{N-1}_{N} \end{bmatrix} \begin{bmatrix} \beta_{n1} \\ \beta_{n2} \\ \vdots \\ \beta_{nN} \end{bmatrix} = \begin{bmatrix} \Gamma_{n0} \\ \Gamma_{n1} \\ \vdots \\ \Gamma_{n(N-1)} \end{bmatrix}. \tag{13}$$

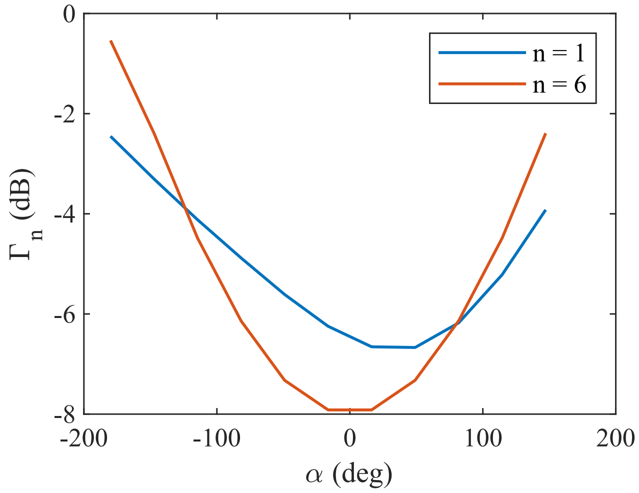

Figure 1. Active reflection coefficient (Γ_n) vs progressive phase (α) for edge element ($n = 1$) and central element ($n = 6$).

The S_{np} terms are then

$$S_{np} = \frac{\beta_{np}}{e^{j(p-n)\pi}} \tag{14}$$

for each value of n and p. The S-parameter matrix can typically be assumed to be symmetrical, i.e. $S_{np} = S_{pn}$.

3 Finite ULA Results

The formulation above was applied to an 11-element linear $\frac{\lambda}{2}$ dipole array above an infinite ground plane and compared to directly simulated S-parameter results. The frequency was 1 GHz and the interelement spacing was $\frac{\lambda}{2}$. Simulations were performed using Altair FEKO.

The simulated active reflection coefficient (Γ_n) versus progressive phase (α) for the central element ($n = 6$) and an edge element ($n = 1$) is shown in Figure 1. It can be seen that for the central element, Γ_n is symmetrical about $\alpha = 0$, while this is not the case for edge elements. To obtain accurate S-parameters, especially for edge elements, a separate Prony model should therefore be solved for each element.

Figure 2 depicts the magnitude and phase of the S_{np} terms for coupling to a central element ($n = 6$). The terms calculated using the Prony method are compared to those determined by direct simulation. Figure 3 shows a similar plot, but for coupling to an edge element ($n = 1$). It is clear that there is excellent agreement between the results calculated using the Prony method and directly simulated results for both central and edge elements.

4 Infinite Array Approximation

As was pointed out above, solving a separate Prony model for each n will give the most accurate results. If array edge

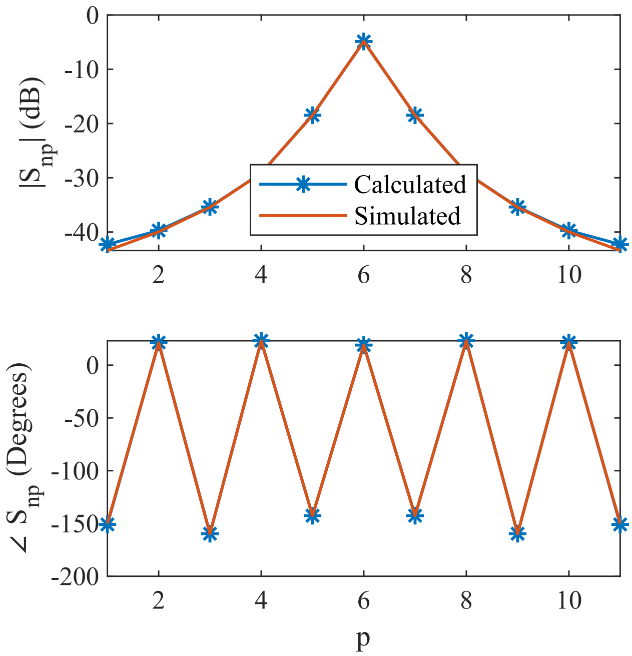

Figure 2. Mutual coupling (S_{np}) between a central element ($n = 6$) in 11-element linear dipole array and all other elements (p). Results are obtained using the Prony method and direct simulation of the S-parameters.

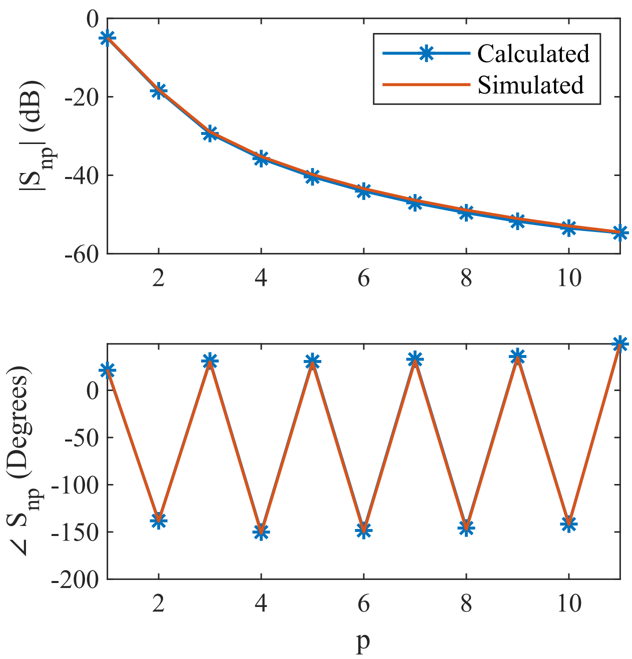

Figure 3. Mutual coupling (S_{np}) between an edge element ($n = 1$) in 11-element linear dipole array and all other elements (p). Results are obtained using the Prony method and direct simulation of the S-parameters.

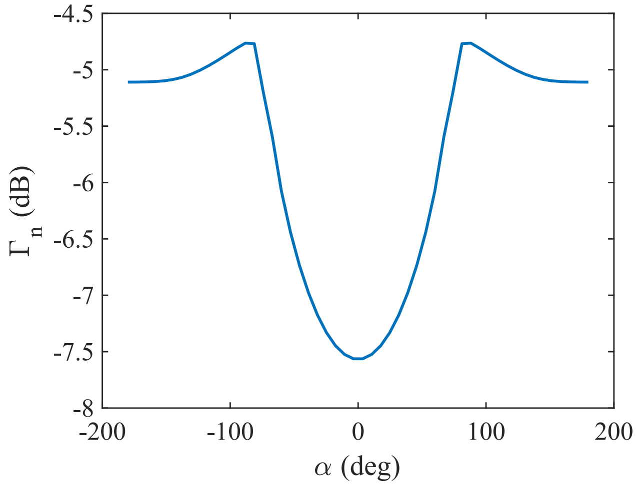

Figure 4. Active reflection coefficient (Γ_n) vs progressive phase (α) for an infinite array.

effects can be ignored, though, the array can be approximated as an infinite array. The calculated S-parameters can then be interpreted as the coupling between N excited elements embedded in an infinite array of match-terminated elements. The assumptions stemming from this approximation will allow the use of a single Prony model to populate the entire $N \times N$ S-parameter matrix.

The assumptions are as follows: since the array is uniform, it is assumed that coupling terms of the same relative offset are equal

$$S_{np} = S_{n+1,p+1} = S_{n+2,p+2} = S_{n+m,p+m}. \tag{15}$$

This also holds for absolute offset

$$S_{np} = S_{n,-p} = S_{n,|p|}. \tag{16}$$

Now, all the necessary relative S-parameters can be solved using the Prony model for $n = 1$, and the remainder of the approximated S-matrix can be populated using the assumptions above.

To illustrate the application of the infinite array approximation to the Prony model, an infinite $\frac{\lambda}{2}$ dipole ULA above a ground plane was simulated. The frequency was set again at 1 GHz and the interelement spacing was 240 mm. The Periodic Boundary Condition functionality of FEKO was used for the infinite array approximation.

The active reflection coefficient of the infinite array is plotted against the progressive phase in Figure 4. The symmetry about $\alpha = 0$ is again evident. This symmetry can be further exploited to halve the number of α values that need to be simulated to populate the Γ_n matrix in equation (13).

After the Prony model was applied to determine the S-parameters of a 51-element array, the results were compared to those of a direct S-parameter simulation of a finite 51-element array.

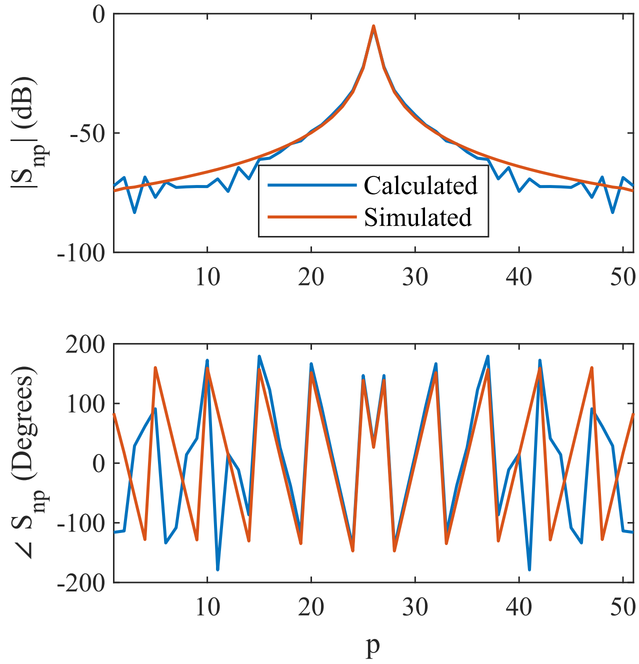

Figure 5. Mutual coupling (S_{np}) between central element ($n = 26$) and all other elements (p) in 51-element linear dipole array. Results are obtained using the Prony method applied to an infinite array approximation and direct simulation of the S-parameters of a finite array.

Figure 5 shows the magnitude and phase of the mutual coupling between a central element ($n = 26$) and all other elements. The results of the infinite array Prony method compares well with the directly simulated results down to magnitudes of ≈ -50 dB, which is negligibly small at any rate.

The results of an edge element ($n = 1$) are shown Figure 6. The values for $p < 10$ are fairly accurate and give a good approximation of the coupling, but it is clear that larger relative offsets are affected by the inaccurate application of symmetry to the Γ_n matrix.

5 Comments

Setting α to $\pm\pi$ will in almost all cases scan the beam to the invisible region, which might not be desired. However, using sample values of α as indicated in equation (6) result in the N base terms $\Phi_{np} = e^{j(n-p)\Delta}$ to be equally spaced on the unit circle in the complex plane. This leads to a perfectly conditioned Vandermonde matrix [7] allowing the system in equation (13) to be solved without any further pre-conditioning.

For the simple dipole example presented in this paper, the application of infinite array approximation had a much longer runtime than the direct simulation of the S-parameters. Altair FEKO's solver is based on the Method-of-Moments (MoM), where the computational time scales as the cube of the number of basis functions. This example still had very few basis functions, so it could still be

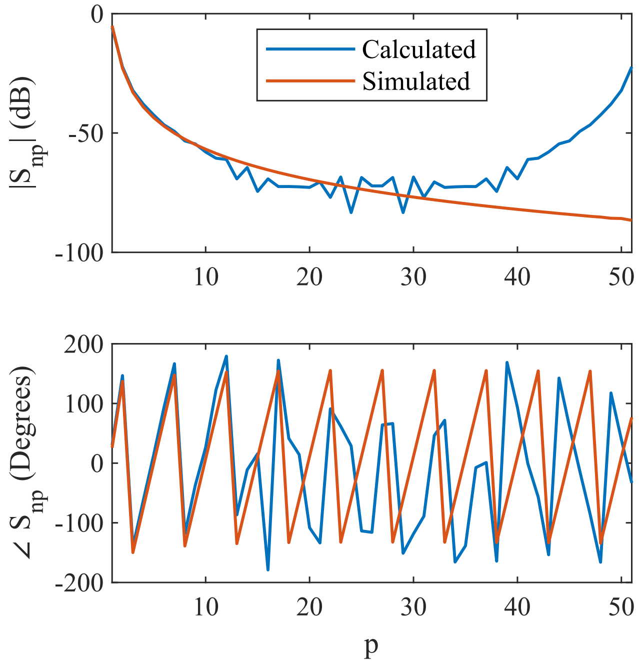

Figure 6. Mutual coupling (S_{np}) between an edge element ($n = 1$) and all other elements (p) in 51-element linear dipole array. Results are obtained using the Prony method applied to an infinite array approximation and direct simulation of the S-parameters of a finite array.

quickly solved. Since only the unit-cell of a single element is solved, the infinite array Prony method will be useful as a simulation method in large arrays with intricate designs or designs with dielectrics that require a large number of basis functions per element.

6 Conclusion and Future Work

Multi-exponential analysis was successfully used to extract the mutual coupling terms of uniform linear antenna arrays from the active reflection coefficient. Results were presented for simulations of a finite array and an infinite array approximation.

Future work will include expanding this model to the uniform planar array case.

The formulation will also be adapted so that it can be used to determine coupling terms in arrays from physical measurements of the embedded element pattern.

7 Acknowledgements

This research has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 101008231 (EXPOWER).

This work is based on the research supported in part by the National Research Foundation of South Africa (Ref Number CSUR23030881506).

References

[1] A. Cuyt and W. shin Lee, "Multivariate exponential analysis from the minimal number of samples," *Advances in Computational Mathematics*, vol. 44, pp. 987–1002, 8 2018.

[2] R. Raal, "Sub-sampled exponential analysis applied to sparse planar antenna array configurations," Ph.D. dissertation, University of Antwerp, 2022.

[3] F. Knaepkens, "Multidimensional exponential analysis: theory and applications," Ph.D. dissertation, University of Antwerp, 2022.

[4] D. Buck, K. F. Warnick, R. Maaskant, D. B. Davidson, and D. F. Kelley, "Measuring array mutual impedances using embedded element patterns," *IEEE Transactions on Antennas and Propagation*, vol. 71, pp. 606–611, 1 2023.

[5] D. Pozar, "A relation between the active input impedance and the active element pattern of a phased array," *IEEE Transactions on Antennas and Propagation*, vol. 51, pp. 2486–2489, 9 2003.

[6] K. F. Warnick, R. Maaskant, M. V. Ivashina, D. B. Davidson, and B. D. Jeffs, *Phased Arrays for Radio Astronomy, Remote Sensing, and Satellite Communications*. Cambridge University Press, 7 2018.

[7] V. Y. Pan, "How bad are vandermonde matrices?" *SIAM Journal on Matrix Analysis and Applications*, vol. 37, no. 2, pp. 676–694, 2016.