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# An h-p Refinement Strategy for Locally Corrected Nystrom Discretization of CFIE on 3D Objects with Sharp Corners

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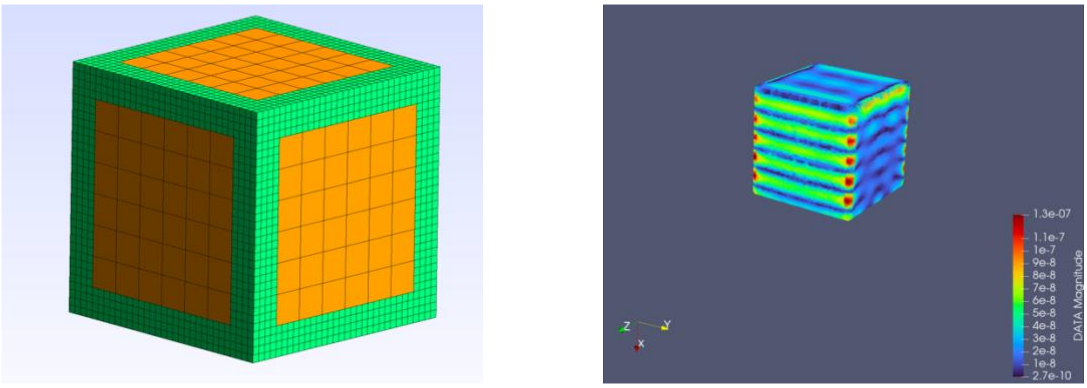
The accurate numerical solution of integral equations on sharp-edge geometries presents a significant challenge, particularly in point-based discretization methods. Traditional approaches often struggle with singularities at edges, leading to reduced accuracy or excessive computational costs. Methods such as the Method of Moments (MoM) provide robust solutions but require specialized basis functions to handle singularities effectively. In contrast, point-based methods, such as Nyström discretization, offer computational efficiency but require careful treatment of sharp edges to maintain accuracy.

In this work, we present a framework for efficiently solving sharp-edge geometries in integral equations without explicitly capturing singularities at the edges. The framework is designed for point-based discretization methods, particularly the mixed-order Locally Corrected Nyström (LCN) method and Chebyshev-based Nyström (CBIE) discretization. A hybrid approach is introduced: low-order solutions with local h-refinement are employed at edge elements to maintain accuracy, while high-order LCN or CBIE methods are applied to non-edge elements. This strategy ensures both accuracy and computational efficiency, mitigating the numerical difficulties posed by sharp edges. The results validate that this framework enhances solution accuracy while maintaining numerical stability in sharp-edge geometries.

The framework is tested on a variety of practical geometries, with a detailed case study performed on a 2m cube at 400 MHz in Fig. 1. In this example, the cube's edge regions (green elements) are assigned second-order LCN with h-refinement to 0.25 of the adjacent smooth patch size, while smooth patches (orange elements) are discretized using seventh-order LCN without additional h-refinement to solve the CFIE problem. The results confirm that this mixed-order approach effectively reduces numerical artifacts and improves solution precision without excessive computational overhead in the edges, especially with the hyper-singularity in the EFIE problem.




(a) Discretization with locally h-refinement for edge elements

(b) Surface current for dipole excitation located at -x axis with z polarization

**Figure 1.** 2m Cube at 400MHz with h-p refinement framework in CFIE

## References

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