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Accurate Calculations of Complex Eigenvalues in Open Prolate Spheroidal Cavities

Elena D. Vinogradova* (1) and Galyna Prytula(2)
(1) Macquarie University, Sydney NSW 2109, Australia; e-mail: elena.vynogradova@mq.edu.au
(2) O. Y. Usikov Institute for Radiophysics and Electronics NASU, Kharkiv 61085, Ukraine;
e-mail: pritula.galina@gmail.com

The eigenfrequencies in closed prolate spheroidal conducting cavities have been exhaustively examined (see, for example, [1]). To our knowledge, the corresponding values for open spheroidal cavities have not been calculated, at least, by rigorous methods, even though open spheroidal shells are a closer match to the shape of some practical microwave devices.

This work partially eliminates the gap in spectral studies of spheroidal shells and may be treated as a first contribution to further studies. The geometry of the open perfectly electrically conducting (PEC) prolate spheroidal cavities investigated is as follows. The first is the rotationally symmetric cavity formed by a single circular aperture opened in the prolate spheroid; the second is the symmetrical structure in which two equal coaxial apertures are opened, the so-called "spheroidal barrel".

The calculation of the relative eigen-wave numbers $\gamma_{snm} = k_{snm}a$, where the imaginary part of the complex number $\gamma_{snm} = \gamma'_{snm} + i\gamma''_{snm}$ is negative ($\gamma''_{snm} < 0$), employs the rigorous Method of Analytical Regularization (MAR) applied to these open spheroidal cavities [2, chp. 7]. The MAR produces regularised solutions to scattering problems with guaranteed prescribed accuracy. This forms the basis for an iterative method for obtaining the complex eigenvalues outlined in [3].

In the first stage of our investigation, we extract the complex eigenvalues γ_{snm} related to the natural oscillations of electrical type TM_{snm}. The cavities are defined by the aspect ratio $q = a/b$ of minor and major axes a,b ($a \leq b$), with aperture size specified by a suitable angle θ_0. Calculations are performed for structures which vary in aspect ratio q from 1 (a sphere) to $q \ll 1$ (a nearly circular cylinder); also, a wide range of aperture size is considered. An accuracy of calculations at the level of 5-6 significant decimal places is achieved.

References

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[2] S. S. Vinogradov, P. D. Smith, and E. D. Vinogradova, *Canonical problems in scattering and potential theory. Part 2: Acoustic and electromagnetic diffraction by canonical structures*, Chapman & Hall/CRC, 2002, 503 pp.

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