Resonant Coupling of a Deep-Subwavelength Particle in a Rectangular Conducting Cavity: Beyond the Jaynes-Cummings Model

Koffi-Emmanuel Sadzi,(1,2) and Yakir Hadad(2)
(1) School of Electrical and Computer Engineering, Tel-Aviv University, e-mail: yakirhad@tauex.tau.ac.il
(2) Current address: Jacob School of Engineering, University of San Diego, USA

In this work, we investigate the resonant interaction between a deep-subwavelength particle and an enclosed, perfectly conducting rectangular cavity. The particle under consideration may be a semiconductor plasmonic nanoparticle, a ferrite particle, or a quantum emitter, such as a Rydberg atom, which emits in the microwave regime. Since metals at microwave frequencies can be accurately modeled as ideal conductors, our study is highly relevant to practical cavity quantum electrodynamics (QED) and wave physics applications. Our primary objective is to determine the joint resonances (eigenfrequencies) of the coupled cavity-particle system without imposing constraints on the coupling strength. Unlike conventional quantum optics models, such as the Jaynes-Cummings model, which considers the interaction of an emitter with a single or a small number of cavity modes, our formulation incorporates the complete coupling dynamics by taking into account all cavity modes. This comprehensive treatment ensures that our model remains strictly causal, in contrast to the commonly used single-mode quantum Rabi model, which has been shown to exhibit causality issues (see, e.g., [1-2]).

From a mathematical perspective, we introduce a novel singularity extraction technique for the local field Green's function $G^{\text{loc}}(\boldsymbol{r})$ inside a cavity. The latter is defined as the field created at the particle location $\boldsymbol{r}$ by backscattering due to the particle's own field. Mathematically it reads $G^{\text{loc}}(\boldsymbol{r}) = G^{\text{cav}}(\boldsymbol{r},\boldsymbol{r}) - G^{\text{fs}}(\boldsymbol{r},\boldsymbol{r})$ where $G^{\text{cav}}(\boldsymbol{r},\boldsymbol{r})$ and $G^{\text{fs}}(\boldsymbol{r},\boldsymbol{r})$ denote the cavity and the free-space Green's functions where the observer is located at the particle location. Namely, the local Green's function consists of subtraction of two different functions sharing identical singularity, having an indefinite $\infty - \infty$ form. How can we calculate this difference using a numerically stable and efficient approach? The talk will focus specifically on this issue, solving the problem in a recursive fashion for a rectangular cavity. The same approach can be adapted for different geometries of the cavity. Once $G^{\text{loc}}(\boldsymbol{r})$ is found, the effective polarizability dyad [3] of the particle in the cavity can be expressed as $\alpha_{\text{eff}} = \left[\alpha^{-1} - G^{\text{loc}}(\boldsymbol{r})\right]^{-1}$ where α denotes the dynamic particle's polarizability in free-space, encapsulating the particle's material and geometry. Then, the collaborative resonance frequencies ω_r of the coupled system are found by setting $\det \alpha_{\text{eff}}^{-1}(\omega_r) = 0$

Our approach enables numerically stable and computationally efficient determination of the eigenfrequencies of the coupled system. Our approach may also find significance in the context of computational electromagnetics, where it can be directly integrated into integral equation-based methods, such as the Method of Moments (MoM), for solving electromagnetic scattering and excitation problems involving inhomogeneity within cavities. Specifically, our singularity extraction method provides a robust framework for computing the self-term in the impedance matrix of the MoM formulation (see, e.g., [3]).

Beyond its theoretical contributions, our formulation offers direct applications in quantum optics, cavity QED, and computational electromagnetics. By extending the conventional understanding of cavity-particle interactions into the strong and ultrastrong coupling regimes while preserving full causality, this work contributes to the accurate modeling of wave phenomena in enclosed environments with subwavelength resonant inclusions.

References

[1] C. S. Muñoz, F. Nori, and S. De Liberato, "Resolution of superluminal signalling in nonperturbative cavity quantum electrodynamics," Nat. Commun. 9 1924 (2018).
[2] J. Larson and T. Mavrogordatos, *The Jaynes–Cummings Model and Its Descendants,* IOP Publishing (2023).
[3] L. Novotny and B. Hecht, *Principles of Nano-optics*, Cambridge Press (2012).
[4] A. F. Peterson, R. Mittra, and S. L. Ray, *Computational Methods in Electromagnetics*, IEEE Press, (1998).