Homogenization of Cylindrical Wire Structures

Or Tayar [(1)], Yarden Mazor [(1)]
(1) School of Electrical and Computer Engineering, Tel-Aviv University, Tel-Aviv, 69780, Israel

Homogenization of sub-wavelength electromagnetic structures is a well-known practice that has been studied for many different types of systems. It is meant to replace an intricate structure with simple, effective parameters that allow its modeling using significantly simplified tools. The same practice is also applied for planar, two-dimensional structures, replacing gratings, subwavelength particle arrays, and small slots with effective surface susceptibility χ_s or surface impedance Z_s [1, 2]. Obtaining the effective parameters normally involves devising a solution for the electromagnetic field scattering problem and performing various approximations to obtain an analytical or semi-analytical expression for the effective parameters. The vast majority of these surface homogenization schemes assume that the structure is an infinite, planar, periodic arrangement of inclusions and applies the ensuing simplifications. However, in many cases, we use such structures in a cylindrical, rather than planar, setup. Several works have tackled these structures, and usually, it is assumed that the same homogenization scheme can be used for the cylindrical case to model the structure as a homogeneous cylindrical sheet.

In this work, we would like to examine more closely in which parameter regimes this assumption is valid, and which cases require a deeper look, and a revision of the homogenization scheme. The basic electromagnetic problem we study is a simple, cylindrical structure - the cylindrical wire grid. N wires are placed on the circumference of a cylinder with radius R, where each wire has a radius of r_0. In the simple case the wires are assumed to be made of a perfect electric conductor PEC, and the problem is studied in TE polarization, with an incident electric field $E^{inc} = E_0 e^{-jkx}\hat{z}$. The same formulation can also be applied to study loaded wires, periodically loaded with impedance Z_L/Δ per unit length. The periodic loading is manifested through a revised wire susceptibility α. This renders the current I in each wire to an impinging local field E_z^L

$$I = \alpha E_z^L \ , \ \alpha^{-1}(\omega) = \frac{Z_L}{\Delta} + \frac{\eta k}{4} H_0^{(2)}(kr_0). \tag{1}$$

with k the wavenumber associated with the environment and η the wave impedance. The current in each wire of the wire array can be derived from

$$\frac{1}{\alpha} I_n - \sum_{m \neq n} G(r_n, r_m) I_m = E_z\,(r_n) \tag{2}$$

with $G(\vec{r}, \vec{r}') = -\frac{j}{4} H_0^{(2)}(k|\vec{r} - \vec{r}'|)$ being the free-space Green's function, and r_n the location of the n'th wire. Once the currents are calculated, an effective representation of the grid array can be derived in terms of surface conductivity. We will show that while in many cases the planar homogenization scheme might suffice, there are discrepancies, which can be accounted for by explicitly considering two important effects. First, the interaction between the wires does not occur on an infinite planar geometry and needs to be properly considered. Second, it is important to consider the spatial nonlocality that could arise, stemming from the nonzero structural curvature. We present a revised scheme to derive the surface conductivity for a cylindrical structure, and examine the results in various operation regimes. We compare the leading term of this scheme to the planar result, map the significance of the nonlocal correction, and look into possible approximations. The results are also validated using full-wave simulations.

References

[1] Senior, T.B. and Volakis, J.L., 1995. Approximate boundary conditions in electromagnetics (No. **41**). Iet.

[2] Tretyakov, S., 2003. Analytical modeling in applied electromagnetics. Artech House.