*URSI-B EMTS 2025, Bologna, 23–27 June*

# Pyramid Elements for Finite Methods

Roberto D. Graglia, Damiano Franzò, Paolo Petrini
Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy
roberto.graglia@polito.it; damiano.franzo@polito.it; paolo.petrini@polito.it

Modern finite methods electromagnetic (EM) solvers model complex geometries using hybrid three-dimensional meshes composed of cells of different shapes, namely a mixture of tetrahedra, hexahedrons (bricks), triangular prisms, and pyramids. In particular, square-based pyramids are used to connect tetrahedral and hexahedral mesh schemes or, in combination with triangular prisms, to glue brick meshes of different densities. Pyramids are also useful when using *h*-adaptive techniques that require iterative refinement of the mesh on a zone-by-zone basis, which is easier if hybrid meshes are allowed [1]. In all cases, to obtain conforming meshes, the quadrilateral and triangular faces of a pyramidal element must match those of the other differently shaped elements that have quadrilateral and triangular faces. In this regard, we note that while there is little to add on how to define and use tetrahedral, prismatic and brick elements for applications of finite methods [1], much remains to be clarified and validated regarding pyramids, for which the literature is more recent, less extensive, less coherent and, at times, discordant (see [2,3,4] and the references therein).

In [3, 4] we have obtained hierarchical curl- and divergence-conforming bases for the pyramid using a very simple multiplicative construction technique, multiplying the lowest-order vector functions by a set of complete scalar polynomials up to a given order. More recently, with the same technique, we also succeeded in obtaining interpolatory higher-order bases for the pyramid. In particular, the interpolatory curl-conforming bases are discussed and tested in [5] while the interpolatory divergence-conforming bases are still unpublished. The multiplicative polynomials used to build the interpolatory bases are linear combinations of scalar Silvester polynomials, given by very simple recursive formulas [1]. The hierarchical bases are instead obtained using multipliers that are Jacobi polynomials associated with more complex recursive formulas since Jacobi polynomials depend on two parameters. Notice that hierarchical bases describe the same vector space described by interpolatory bases because the basis functions of the first set are linear combinations of the second, and vice versa. For *h*-adaptive techniques, one can therefore use hierarchical or interpolatory bases without major distinctions. On the contrary, to implement *p*-adaptive techniques it is more convenient and simpler to use hierarchical bases [1]. The pyramid’s curl-conforming bases (both interpolatory and hierarchical) have been tested in [5], [6] studying EM resonators of different shapes, even with curved walls.

The oral presentation will focus on the main difficulties encountered in developing the pyramid's bases, summarizing the technique used to build hierarchical and interpolatory bases, and the *scalar* shape functions suitable for describing pyramids with curved walls. Several numerical results for resonant cavities will also be presented to validate the effectiveness of using these bases in computational electromagnetics problems.

## References

[1] R.D. Graglia, and A.F. Peterson, Higher-order Techniques in Computational Electromagnetics, SciTech Publishing/IET, Edison, NJ, 2016.
[2] F. Fuentes, B. Keith, L. Demkowicz, and S. Nagaraj, “Orientation embedded high order shape functions for the exact sequence elements of all shapes,” Computers & Mathematics with Applications, vol. 70, pp. 353-458, 2015.
[3] R. D. Graglia and P. Petrini, “Hierarchical curl-conforming vector bases for pyramid cells,” IEEE Trans. Antennas Propagat., vol. 70, no. 7, pp. 5623-5635, July 2022, doi: 10.1109/TAP.2022.3145430.
[4] R. D. Graglia, “Hierarchical Divergence-Conforming Vector Bases for Pyramid Cells,” in IEEE Transactions on Antennas and Propagation, vol. 71, no. 12, pp. 9334-9343, Dec. 2023, doi: 10.1109/TAP.2023.3241443.
[5] R. D. Graglia, D. Franzò, and P. Petrini, “Interpolatory Curl-Conforming Vector Bases for Pyramid Cells,” in IEEE Transactions on Antennas and Propagation, (Early Access), doi: 10.1109/TAP.2025.3532115.
[6] R. D. Graglia and P. Petrini, “Assessing Curl-Conforming Bases for Pyramid Cells," in IEEE Journal on Multiscale and Multiphysics Computational Techniques, vol. 9, pp. 20-26, 2024, doi: 10.1109/JMMCT.2023.3333563.