Key Points in Electromagnetic Inverse Scattering Problems: Scattered Field Dimension

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Inverse scattering amounts to reconstruct an unknown scattering object from scattered field measurements taken outside the object when it is probed by known incident fields. As such, it is relevant in all wave disciplines like Quantum Physics, Optics and Electromagnetics [1]. Besides, the even increasing relevance of inverse scattering in countless applicative contexts has triggered the research and it is continuing to push the scientific community to look for advancements and a better understanding of the key factors characterizing the problems and ultimately the achievable performance.

Across the years a large body of results have been spread in literature and countless reconstruction algorithms have been proposed: deterministic, stochastic and more recently Machine Learning based approaches [2]. Also, approximate linear, quadratic and bilinear [3], [4] problem formulations have been explored as well.

The main question is to dominate the very features of electromagnetic inverse scattering problems, i.e., non-linearity and ill-posedness, which lead to a very difficult problem. Indeed, besides the computational cost, since reconstruction is generally cast as the minimization of a non-quadratic non-convex functional, reliability issues arise owing to the occurrence of false solutions corresponding to the local minima of the function [5].

Though different algorithms can, in principle, performs differently, a clear understanding of what can and cannot be achieved depends on the mathematical properties of the scattering equations, the level of noise, etc. In this regard, key points that must be considered are the number of degrees of freedom of the problem [6] (related to the intrinsic filtering propagator entails), the ratio between the number of "independent" data and unknowns [7] (which affects the curvature of the set data belong to and hence is crucial for local minima occurrence), and how they are influenced by the employed configuration. A priori information about the scatters' supports, the degree of smoothness and symmetries of the contrast function, etc., when available, must enter the picture as well. Indeed, a crucial question is to determine how a priori information impacts on the mentioned key points and correspondingly helps in tailoring the reconstruction procedure. Besides, further related important theoretical and practical questions are the way scattered field should be "sampled" [13] and the choice of the functional subspace for representing the unknown. The first one entails findings out how many and to optimally deploy sensors in order to reach the available number of degrees of freedom. This is clearly relevant from problem stability and above all to set the cheapest possible configuration. The second question entails determining a suitable representation base that matches the filtering introduced by propagation and a priori information about the contrast, which in turn crucially affects the local minima problem.

In this contribution, we trace a path across the research activities we have developed on the key points mentioned above. The journey will touch linear and non-linear formulations and in particular focuses on quadratic scattering models where results concerning the dimensionality of the scattered field data space and the data/unknowns ratio (which affects the occurrence of local minima) will be provided for the case pedagogical case of a 2D scalar geometry.

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