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Preliminary results on the number of communication channels within in-homogenous medium

Mario Del Prete[(1)], Maria Antonia Maisto[(1)], and Raffaele Solimene[(1)]
(1) Università degli Studi della Campania, Aversa (CE), Italy

Abstract

This paper addresses the problem of estimating the information content that can be sent by a source to an observation domain. In particular, the connection between information content and the Number of degrees of freedom is discussed. Using arguments based on the warping variable, the information content can be easily estimated by counting the number of samples that fall in a fixed measurement aperture. Finally, it is shown that the inhomogeneous background increases the information content compared to a homogeneous scenario.

1 Introduction

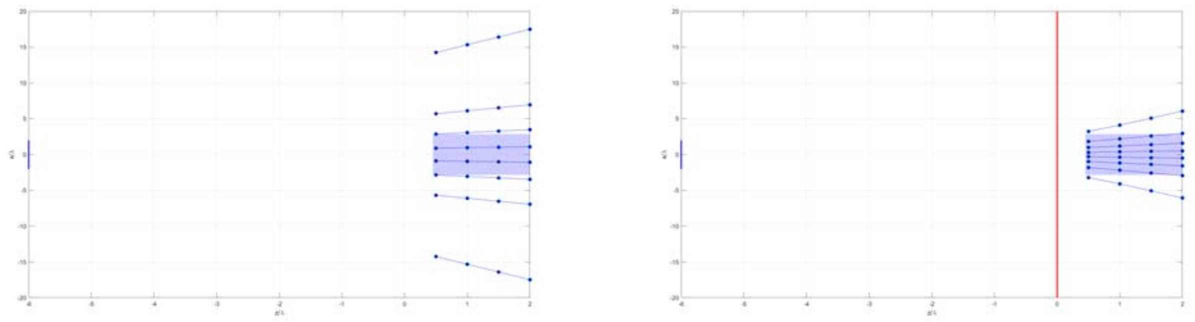
The number of degrees of freedom (NDF) of the radiated fields plays a crucial role in characterizing both forward and inverse problems in electromagnetics [1]. Looking at the problem from the field space, the NDF represents the number of significant and independent parameters needed to represent the field with a given degree of accuracy and also identifies the essential dimension of the field that can be radiated by a source defined on compact support [2]. It is related also to the achievable resolution [3] and in particular to the information content estimation. By interpreting the radiation and scattering phenomenon as a way to propagate information from an investigation to an observation domain, the NDF is linked to the question of estimating the number of communication channels that significantly connect the investigation and the measurement volumes [4]. This point of view is of fundamental importance in space-time wireless systems and in particular to multiple-input multiple-output (MIMO) communication systems [5] Furthermore, in the framework of Kolmogorov's theory of ε-entropy and ε-capacity, the NDF is linked to a measure of the number of ε-distinguishable messages in presence of noise data [4],[6]. Note that the NDF is always finite even when the observation domain is unbounded and depends only on the measurement configuration parameters [7]. Moreover, it has been demonstrated that NDF is associated with the bandlimited representations of the field with a finite and non-redundant sample number [8]. This relationship establishes a correlation between sampling and the information content of the data. To this end it was shown through an appropriate warping transformation of the observation variable, that the electromagnetic field could be efficiently represented, with a number of samples equal to NDF, properly distributed over the measurement aperture in [9]-[12]. In this paper, this strategy is used to estimate the information content and to show that the introduction of inhomogeneity in the medium positively impacts the distribution of samples in space by increasing the information that can be conveyed from the source to a fixed observation domain.

2 Number of Degrees of Freedom and Information Content

Let $\mathbf{G}$ be the compact radiation operator that links the functional space of the sources spaces $\mathbf{X}$ with that of the field $\mathbf{Y}$

$$\mathbf{G} : x \in \mathbf{X} \rightarrow y \in \mathbf{Y} \tag{1}$$

Since the propagation phenomenon can be seen as a way to propagate information from $\mathbf{X}$ to $\mathbf{Y}$, the operator $\mathbf{G}$ acts like a communication channel. In a communication system, information is associated to different messages. Consequently, roughly speaking, the maximum amount of information conveyed by a communication channel is equal to the maximum number of different messages which are distinguishable also when the observed data are affected by uncertainty ε [13]. In literature, a way to quantify the information content conveyed through this channel in the presence of noise level ε can be expressed in terms of the Number of Degrees of Freedom (NDF). In short, the NDF is the number of independent parameters required to represent the radiated field with a given degree of accuracy. The NDF also is linked to the sampling theory and in particular to the representation of the electromagnetic field In [8], it was shown that the field radiated by a bounded source is ”essentially” band-limited and can be represented(with a given degrees of accuracy) by a finite and non-redundant number of samples equal to the NDF. This means that from the information theory point of view, each sample is associated with independent information transmitted along the communication channel. Another way to quantify the information content is based on Kolmogorov's ε-packing metric, which introduces a framework in which information is topologically characterized without relying on probabilistic assumptions, in contrast to classical Shannon's information theory [6]. Denote as $B(0,\gamma) = \{x \in \mathbf{X} : \|x\| \leq \gamma\}$ as the hyper-sphere of radius γ, and let $\mathbf{G}(B(0,\gamma))$ its image under $\mathbf{G}$ where γ is the bound of the source energy. To quantify information, define the ε-capacity, $C(\varepsilon)$, as the maximum number ε-distinguishable points that belong to



(a) (b)

Figure 1. Distribution of samples in the spatial domain obtained by the warping strategy and calculated for different distances between the observation domain and the source domain (blue interval on the left). The geometrical parameters are $X_s = 2\lambda$, $X_o = 2.85\lambda$ (purple strip). Panel a: Homogeneous background(free space). Panel b: Inhomogeneous background. In this case, the vertical red line indicates the interface between the left medium($\varepsilon_l = 16$) and the right medium (free space)

$\mathbf{G}(B(0,\gamma))$. This measure reflects the topological information transmitted by the operator. Due to the compactness of the radiation operator $C(\varepsilon)$ is finite. For practical purposes, Kolmogorov's ε-entropy $H(\varepsilon)$, which approximates $C(\varepsilon)$, serves as an alternative measure. Specifically, $H(\varepsilon)$ represents the logarithmic count of ε-covering sets required to span $\mathbf{G}(B(0,\gamma))$. The latter forms a hyper-ellipsoid, with axes scaled by singular values σ_n of $\mathbf{G}$. The number of balls of diameter 2ε required to cover the ellipsoid is:

$$H(\varepsilon) = \log_2 \prod_{n=0}^{N_\varepsilon} \frac{\gamma\sigma_n}{\varepsilon} = 2\sum_{n=0}^{N_\varepsilon} \log_2 \frac{\gamma\sigma_n}{\varepsilon} \tag{2}$$

where $N_\varepsilon = \max\{n > 0 : \sigma_n \geq \varepsilon/\gamma\}$,σ_n is the n-th singular values of $\mathbf{G}$ and ε/γ is the noise to signal ratio. Basically N_ε is the NDF and then the latter equation establishes the connection between the Kolmogorov information theory and the NDF. While the two concepts are related, they are not equivalent. As highlighted in (2), once the signal-to-noise ratio γ/ε is fixed, the information content is related to the behavior of the singular values of the operator $\mathbf{G}$. So in principle, it is required to compute the singular spectrum of the radiation operator. Since, in many situations, the dynamic of singular values before the knee (occurring at NDF) is neglected, $H(\varepsilon)$ is proportional to NDF. Accordingly, in the next section the information content is quantified as the NDF.

3 Information content estimation

This section addresses the problem of estimating the amount of information conveyed from a source supported on a finite interval $SD = [-X_s, X_S]$ to a finite observation domain parallel to SD and located in non-reactive zones. Consider this prototype configuration in the free-space and assume to transmit a certain amount of information content. It is clear that the latter cannot be larger than the number of degrees of freedom when the field radiated by the source is observed over an unbounded domain. This number is equal to

$$\mathrm{NDF}_\infty = 4X_s/\lambda + 1 \tag{3}$$

where λ is the wavelength. Accordingly, the source can send $NDF_B \leq NDF_\infty - 1$ information. As discussed in the previous section, there is a connection between appropriate field sampling locations and the NDF and hence the information content. So for our purpose is necessary to find the distribution of these NDF sampling points. Such a point research can be performed employing the so-called warping strategy. It was shown in [9] that by introducing an appropriate observation variable transformation, the field is a bandlimited function in the so-called warping variable and that can be represented with a number of samples equal to NDF in the warping domain. The warping transformation in the case of a homogeneous background [9] is as follows

$$\xi(x_o) = \frac{k}{2}\left(\sqrt{(x_0 + X_S)^2 + z_0^2} - \sqrt{(x_0 - X_S)^2 + z_o^2}\right) \tag{4}$$

where k is the wavenumber and z_o the distance beetween SD and OD. Then, fixing the information content we want to send in terms of NDF_B or equivalently the warping samples at Nyquist rate, we can obtain the position x_{om} in the actual observation variable by solving this equation

$$\xi(x_{om}) = m\pi \tag{5}$$

with $m \in \{1, ..., NDF_B\}$. Once the distribution of samples is obtained, this fixes the size of an aperture OD_B on which the information content sent by the source is equal to NDF_B. To fix the ideas, consider a representative example. It is assumed that $X_s = 2\lambda$ and $NDF_B = 8$. The warping strategy is applied in order to find the distribution of the sampling points by solving the equation (5) for different values of $z_o \in \{6.5\lambda, 7\lambda, 7.5\lambda, 8\lambda\}$. Fig. 1a shows the curves that describe the measurement point distribution as the distance z_o between OD and SD varies. It is important to note that, due to the non-linear nature of the warping transformation, the sampling step is non-uniform along the aperture. In particular, this is smaller in the center and becomes larger moving towards the edges. Furthermore, as z_o increases, an ever larger OD_B aperture is required to transmit the same amount of information. It is evident that in the scenario where a fixed aperture $OD \subseteq OD_B$ is employed, the information content transmitted by the source on it will be less than NDF_B and will be equal to the number of samples falling within OD. In fact, in fig.1a there is a purple strip corresponding to an $OD = [-2.85\lambda, 2.85\lambda]$ that shows this case. By counting the number of samples that fall within this strip, it results that when the size of the source and observation domain is fixed, the information that can be sent to OD tends to decrease as the distance between the source domain and the observation domain increases. At this point, it is natural to ask whether it is possible to transmit the same amount of NDF_B information using an aperture smaller than OD_B assuming z_o remains constant. This can be achieved by modifying the distribution of the samples, particularly by concentrating them toward the center of the aperture. This can be done by introducing an inhomogeneity in the medium. Consider again the same prototype configuration of the previous section but now the background medium consists of two homogeneous non-magnetic half-spaces separated by a planar interface at $z = 0$, with their dielectric permittivity and wavenumbers being ε_r, k_r for the right half-space and ε_l, k_l for the left one with $k_l > k_u$. The source now is in the left space supported on SD far z_s to the interface. It is assumed that the left medium is electromagnetic denser than the right one. Let me highlight that the warping strategy is completely general and that can be applied whatever is the background medium [14]. The warping transformation in the in-homogeneous case [15] is modified as

$$\eta(x_o) = \frac{k_r}{2}[n\sqrt{(f_1(x_o)+X_S)^2+z_s^2}+\sqrt{(x_o-f_1(x_o))^2+z_2^2} \\ -n\sqrt{(f_2(x_o)-X_S)^2+z_s^2}-\sqrt{(x_o-f_2(x_o))^2+z_2^2}] \tag{6}$$

where $n = \sqrt{\varepsilon_l/\varepsilon_r}$ is the refractive index,$|z_s|$ the distance between SD and the interface, $x_m(x_o, x, z_s)$ is the refraction point at the half-space interface accordingly to the Snell law, $f_1(x_o) = x_m(x_o, -X_S, z_s)$ and $f_2(x_o) = -f_1(-x_o)$. Consider the same source $X_s = 2\lambda$ of the previous example with $\varepsilon_l = 16$, $\varepsilon_r = 1$ and $z_s = -6\lambda$. We want to convey the same information content $NDF_B = 8$ as before. Accordingly, to the warping strategy, the distribution of the spatial samples is obtained by solving the equation $\eta(x_o l) = l\pi$ for $l \in \{1, ..., NDF_B\}$. Fig. 1b shows the distribution of samples for z_o which varies in the same range as the previous section in order to make a fair comparison. By comparing fig. 1a and fig.1b, the role of background is clear. Firstly, there is the same trend as in the homogeneous case: As the distance z_o increases, a larger aperture is required to transmit the same information content. But the most important aspect is that for the same z_o, NDF_B information can be transmitted through a smaller aperture size than in the homogeneous case. Moreover, with the same observation domain as in the previous case $X_o = 2.85\lambda$ (purple strip in the figure), the number of samples falling in OD is greater in the inhomogeneous scenario than in the homogeneous one. Basically, the background increases the information content on OD.

4 Conclusion

In this paper, the problem of estimating the amount of information that a source can send on a fixed aperture has been studied. Thanks to the warping strategy, the information content has been estimated by counting the number of samples that fall within a fixed observation domain. Finally, it has been shown that the information content can be increased by introducing inhomogeneity in the background medium.

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