

Buried target localization using MUSIC algorithm and contactless measurement platforms

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Abstract

The problem of detecting and localizing point-like targets buried in a half-space medium from near scattered field measurements collected by a contactless platform under a multi-monostatic configuration is addressed. Under these measurement conditions, super-resolving algorithms such as MUSIC cannot be applied because of the data matrix is not full rank. In this contribution we propose a processing chain in which data rank is restored by working in the wavenumber domain, so that MUSIC can be safely applied without losing its high resolution features. Achievable performance are assessed via numerical examples for a 2D scalar geometry.

1 Introduction

The issue of target localization, along with related challenges such as direction of arrival (DOA) estimation and time of arrival (TOA) determination, is a key and longstanding research topic that has generated extensive studies in recent years. This importance stems from its relevance across a wide range of practical applications, including radar and sonar imaging, wireless communications, astronomy, microwave sensing, and many others [1]–[6].

This paper focuses on subsurface imaging, a field for which numerous reconstruction algorithms have been proposed and explored [7, 8]. Specifically, we tackle the problem of localizing an unknown number of point-like scatterers (i.e., small targets compared to the wavelength) embedded in a homogeneous half-space medium [9]. To approach this, we consider a two-dimensional scalar model with a two-layered background medium. The scatterers are situated in the lower half-space, while the scattered field is measured by a uniform linear array placed in the upper half-space, in the near-field region relative to the buried targets and at a given distance from the air-soil interface. The incident field is modeled as the one radiated by filamentary sources with a specified frequency bandwidth and TM polarization.

It is well-established that the resolution limits of back-propagation algorithms, and similar techniques, are fundamentally tied to the size of the measurement aperture and the frequency band used [10, 11]. These limits are

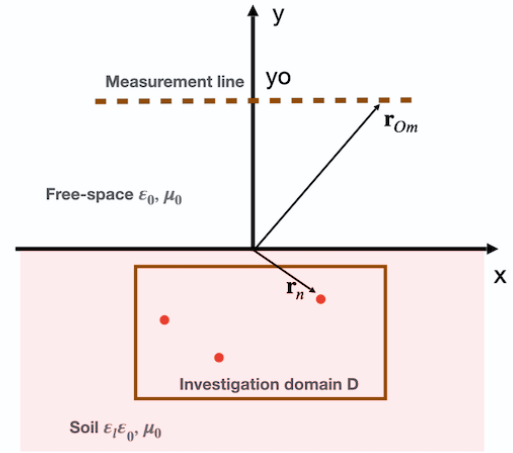


Figure 1. Schematic representation of the considered 2D scattering scene.

commonly referred to as Rayleigh limits [12]. To overcome these resolution limits, target localization is performed using the MUSIC (Multiple Signal Classification) algorithm [13], which is chosen for its ability to achieve super-resolution. MUSIC is a subspace projection method based on the data correlation matrix. However, for the measurement setup at hand and due to the single snapshot acquisition procedure, the data correlation matrix suffers from rank deficiency, leading to a significant decline in the performance of MUSIC [14]. To recover the performance, a rank recovery procedure is necessary. Traditional decorrelation methods typically assume that the matrix propagation operator follows a Vandermonde structure [15]. While this condition holds under far-field propagation or Fresnel zone [16], it is not valid in the near-field setup and in an inhomogeneous medium. As a result, these methods cannot be directly applied. To address this challenge, we demonstrate that by preprocessing the scattered field data, it is possible to reorganize the data in such a way that the Vandermonde structure is effectively restored. This is done by transforming the data into the wavenumber domain [14], rather than working directly in the spatial-frequency domain. Subsequently, a standard smoothing procedure can be applied, and the MUSIC algorithm can then be used to detect and localize the targets.

The paper is structured as follows: Section II outlines the mathematical model of the scattering problem. In Section III, we describe the processing procedure, consisting of a pre-processing step aimed to rank recovery and the application of the well-known MUSIC algorithm. Section IV presents numerical results, which demonstrates the validity of the proposed approach in the considered subsurface localization application.

2 Mathematical model

With reference to Fig.1, let us consider the 2D scalar scattering problem, z being the axis of invariance, occurring in a background medium consisting of two half-spaces, separated by a planar interface at $y = 0$. The lower half-space is modeled as soil, whose dielectric permittivity is denoted as ϵ_l ; the upper half-space is free-space and its dielectric permittivity is denoted by ϵ_0 . Both the half-spaces are non-magnetic and the corresponding magnetic permeability is that one of the vacuum, μ_0 .

N point-like scatterers are buried in the soil within the investigation domain $D : [-X_D, X_D] \times [-Y_D, 0)$. The target positions are $\mathbf{r}_n = (x_n, y_n)$, for $n = 1, 2, \dots, N$. The scattering scene is illuminated by a filamentary source and the scattered field is collected at the same positions as the source while the latter move in order to synthesize the measurement aperture, which is located at the distance y_O from the separation interface. M equispaced measurement positions are considered. In particular, the probing and measurement points are assumed being the same and denoted as $\mathbf{r}_{Om} = (x_{Om}, y_O)$, with $m = 1, 2, \dots, M$ and $-X_O \leq x_{Om} \leq X_O$. The frequency run within the working band $\Omega_f = [f_{min}, f_{max}]$. More in details, the scattered field is collected in near-field region for a discrete set of frequencies f_l , with $l = 1, 2, \dots, L$, by uniformly sampling Ω_f . The number of frequencies and measurement points are determined according to the sampling criterion reported in [17].

According to the Born approximated model [18], the scattered field can be expressed as

$$E_S(\mathbf{r}_{Om}, f_l) = \int_D G(\mathbf{r}_{Om}, \mathbf{r}, f_l) O(\mathbf{r}) E_i(\mathbf{r}, f_l) d\mathbf{r} \quad (1)$$

where $G(\mathbf{r}_{Om}, \mathbf{r}, f_l)$ is the Green function pertinent to the considered two-layered background medium, E_i is the incident field and $O(\mathbf{r})$ represents the object function. In particular, for the case of point-like targets, (1) becomes

$$E_S(\mathbf{r}_{Om}, f_l) = \sum_{n=1}^N G(\mathbf{r}_{Om}, \mathbf{r}_n, f_l) O_n E_i(\mathbf{r}_n, f_l) \quad (2)$$

where now O_n are the target scattering coefficients.

Notably, the inversion of (2), allows us to obtain target localization.

3 Processing strategy

When dealing with single snapshot data, as the case of our interest, the MUSIC algorithm loses its super-resolution power due to the fact that the data correlation matrix is rank deficient.

To cope with this issue, we introduce a pre-processing stage aiming at recovering the rank, even in near-field configuration. Specifically, the first step is to transform the data in the spectral wavenumber domain. Hence, the scattered field is Fourier transformed (with respect to x_O) for each fixed frequency f_l , so that (2) becomes, in the wavenumber domain,

$$\hat{E}_S(k_x, f_l) = \alpha(k_x, f_l) e^{-jk_{0y}y_O} \sum_{n=1}^N O_n e^{jk_x x_n + jk_y y_n} \quad (3)$$

where k_x , $k_y = k_l + \sqrt{k_l^2 - k_x^2}$ and $k_{0y} = \sqrt{k_{0l}^2 - k_x^2}$, are the spatial frequencies in the wavenumber domain, and $\alpha(k_x, f_l)$ takes into account the transmission coefficient (across the separation interface) as well as the Fourier transform (i.e., the plane wave spectrum) of the incident field and the Green Function. In particular, k_l and k_{0l} are the wavenumbers, at the l -th frequency, in the lower and upper half-spaces respectively. While k_x is already uniformly sampled, k_y is not and so an interpolation and re-sampling of k_y is necessary to restore a Vandermonde structure of the spectral matrix and so making it possible the application of smoothing algorithms [19], which in turn allows us to restore the rank of the data matrix.

After doing that, it is possible to apply the MUSIC Algorithm, according to which the localization of the targets is achieved by building up the following pseudospectrum indicator

$$\Phi(\mathbf{r}) = \frac{1}{\|\mathcal{P}_{\mathcal{N}} \text{vect}\{\mathbf{A}(\mathbf{r})\}\|^2} \quad (4)$$

where $\text{vect}\{\mathbf{A}(\mathbf{r})\}$ is the vectorization of the *steering matrix*, with $\mathbf{r} = (x, y) \in D$ being trial position and $\mathcal{P}_{\mathcal{N}}$ denotes the projector operator onto the noise subspace. The indicator peaks when $\mathbf{r}$ coincides with one of the actual target positions.

4 Numerical results

In this section we present few examples in order to check the proposed method. In particular, the analysis focuses on two-points resolution.

The considered bandwidth is $[0.5 - 1.1]$ GHz. The scattered field is collected at $y_O = 8\lambda_m$ (λ_m being the wavelength in correspondence of center frequency and refers to

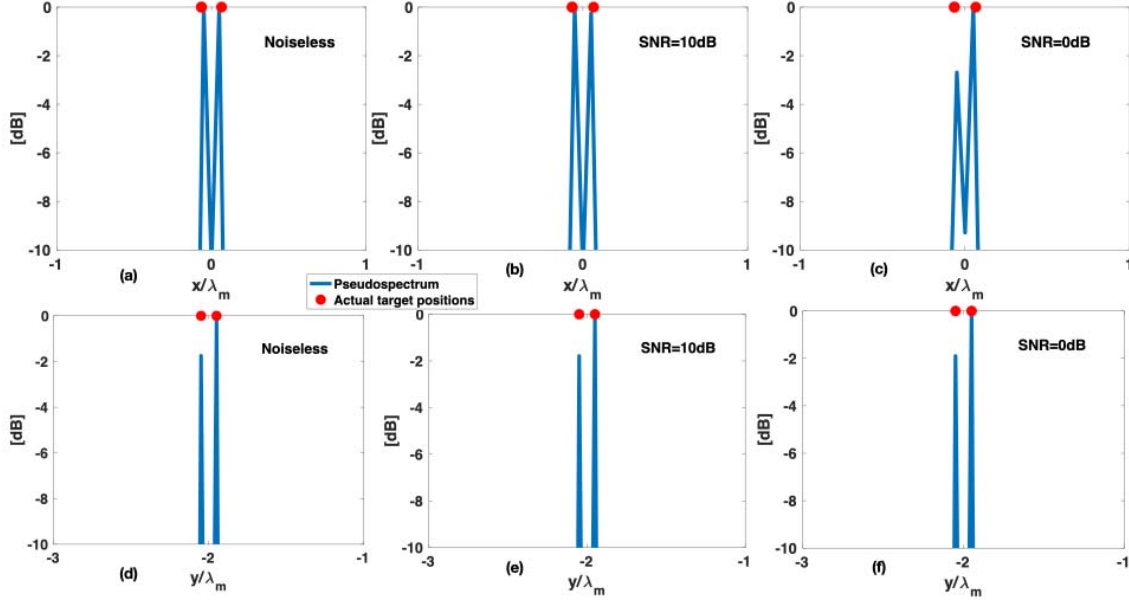


Figure 2. Two-point scatterer resolution analysis in the cross range (first row) and along the range (second row) for different SNR. The red dots highlight the actual targets' locations.

free space) with respect to the separation interface located at $y_{in} = 0$, over the observation line $D_O = [-5\lambda_m, 5\lambda_m]$. The data are taken uniformly according to the sampling step mentioned above [17]. The investigation domain is $D = [-4\lambda_m, 4\lambda_m] \times [-4\lambda_m, 0]$, hence buried in the lower half-space, which in turn has relative dielectric permittivity $\epsilon_l = 9$. In particular, for cross range resolution, we consider the two targets located at the same depth $y = -1\lambda_m$ and separated along x by $0.13\lambda_m$. Instead, for range resolution, the targets are located at $x = 0$ and separated along y by $0.1\lambda_m$. For both cases, the target positions are chosen in order to stay below the resolution limits dictated by the measurement aperture size and frequency bandwidth. The analysis is carried out by varying the signal to noise ratio (SNR) and the corresponding results are shown in Fig.2. The examples show the good performance of the proposed method. In particular, the described procedure allows very well distinguish the two scatterers that are much closer than the classical resolution limits, also for low SNR.

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