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# Classical Photons and the Transition from Classical to Quantum Wave Theory

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It can be proven that a current source of finite energy in a finite region of space cannot radiate a localized electromagnetic pulse with *nondecaying* energy to an infinitely large distance from the source [1], [2, sec. 5.4.4]. However, for limited times and travel distances, classical electromagnetic wavepackets can be excited in free space that mimic photons in the limited sense that they can have a well-defined center frequency and a nearly constant energy concentrated within a small volume (as small as about a cubic center-frequency wavelength).

If this minimum-size (cubic-wavelength) quasi-monochromatic wavepacket is the closest that we can get to a classical representation of a photon, it's cubic-wavelength volume possibly can predict the approximate number of photons per unit volume required for a quantum electrodynamic field to be treated classically. Less than this number of photons per unit volume, quantum scattering by individual photons may be required to accurately predict the scattering of incident electromagnetic fields [3, sec. 8.6.5], [4]. Equating the average energy density $n_{ph}$ of the photons to the average energy density in the classical plane-wave fields $(E_0, H_0)$, one gets

$$n_{ph}\hbar\omega_0 = \frac{1}{4}(\varepsilon_0 E_0^2 + \mu_0 H_0^2) = \frac{1}{2}\varepsilon_0 E_0^2 \quad \text{or} \quad n_{ph} = \varepsilon_0 E_0^2/(2\hbar\omega_0). \tag{1}$$

The average cubic-grid distance between the photons is $d_{ph} = 1/(n_{ph})^{1/3}$. If this average photon separation distance is too large, then classical electromagnetic field analysis needed, for example, to determine the scattering of a single electron in a plane wave (Thomson scattering), may not suffice and a quantum approach to scattering of individual photons bouncing off the electron (Compton scattering) may be required.

The critical separation distance, beyond which classical field theory may fail and quantum theory may be required, can be determined from our above classical model of a photon as the smallest possible quasi-monochromatic electromagnetic wavepacket that can propagate in free space. The linear dimensions of this minimum-size free-space classical quasi-monochromatic wavepacket with well-defined center frequency $\omega_0$ is found, as mentioned above, to be approximately one wavelength $\lambda_0$. Thus, these classical photons will create an approximately uniform classical electromagnetic field in free space if $d_{ph} \lesssim \lambda_0/4$ such that there is appreciable overlap of the wavepackets. Consequently, it seems reasonable to assume that the inequality $d_{ph} \gtrsim \lambda_0/4$ gives the criterion for leaving the homogeneous continuum regime of the classical Maxwellian electromagnetic fields and for encountering significant quantum scattering effects produced by the individual incident photons. This inequality can be re-expressed as

$$n_{ph} \lesssim \frac{64}{\lambda_0^3} \quad \text{or from (1)} \quad \frac{\varepsilon_0 E_0^2}{2}\left(\frac{\lambda_0}{4}\right)^3 \lesssim \hbar\omega_0 \tag{2}$$

which says that quantum scattering theory, rather than classical electromagnetic-wave theory, may be required if the classical electromagnetic-field energy density in a quarter wavelength cubed becomes less than about the photon energy $\hbar\omega_0$. Conversely, if

$$\frac{\varepsilon_0 E_0^2 \lambda_0^3}{2} \gtrsim 64\hbar\omega_0 \quad \left(n_{ph} \gtrsim \frac{64}{\lambda_0^3}\right) \tag{3}$$

then the ensemble of photons can be treated as a classical Maxwellian electromagnetic field. For a given energy density, classical field behavior occurs for frequencies $\omega_0^4 \lesssim 2c^3\varepsilon_0 E_0^2/\hbar$. It is especially noteworthy that the inequality in (3) agrees well with the condition obtained from quantum electrodynamics by Berestetskii, Lifshitz, and Pitaevskii for the "averaged [quantum-electrodynamic] field to be quasi-classical" [4, eq. (5.2)], namely $\varepsilon_0 E_0^2 \lambda_0^3 \gg 2\pi^2\hbar\omega_0$.

## References

[1] T.T. Wu, "Electromagnetic missiles," *J. Appl. Phys.*, vol. 57, pp. 2370–2373, April 1985.

[2] T.B. Hansen and A.D. Yaghjian, *Plane-Wave Theory of Time-Domain Fields: Near-Field Scanning Applications*, New York: IEEE/Wiley, 1999.

[3] A.D. Yaghjian, *Relativistic Dynamics of a Charged Sphere; Updating the Lorentz-Abraham Model*, 3rd ed., New York: Springer, 2023.

[4] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, *Quantum Electrodynamics*, 2nd Ed., Oxford: Pergamon, 1982.