Addressing Spurious Modes in Reduced Order Models for Efficient Electromagnetic Scattering and Metasurface Design

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The time-harmonic Maxwell equations describe the behavior of electromagnetic fields in frequency domain analysis and are often reformulated as a system of two coupled equations for the electric and magnetic fields. When solving these equations numerically using the finite element method, the choice of finite elements is crucial. An inappropriate selection can result in the appearance of spurious modes, which are non-physical solutions that do not satisfy the original governing equations. While they satisfy the curl equations, they typically violate the divergence constraints, leading to unreliable results. The use of edge finite elements, as introduced by Nédélec [1], is a robust and well-established approach to mitigate this issue [2], as these elements naturally enforce the curl-conformity required to suppress spurious modes.

Solving the time-harmonic Maxwell equations with the finite element method often involves a high computational burden, especially for large-scale or parametrized problems. To address this issue, model order reduction techniques have emerged as powerful alternatives, offering significant reductions in computational cost while preserving the accuracy of the solution [3]. Reduced order models (ROMs) project the high-dimensional problem onto a carefully constructed reduced space, capturing the key dynamics of the system but considering far fewer degrees of freedom. This reduction enables much faster simulations without compromising accuracy, making ROMs especially valuable for the electromagnetic analysis of different structures.

However, using a reduced space to approximate the solution can introduce spurious modes [4, 5], as the ROM solution may not satisfy the divergence constraints due to insufficient information from the high-dimensional problem. To overcome this issue, the formulation of the problem proposed by Kikuchi [6] has proven effective. This approach explicitly incorporates the divergence equation into the numerical formulation, ensuring that the full set of Maxwell equations is satisfied and eliminating the possibility of spurious solutions.

In this talk, we present a reduced order model (ROM) for the finite element approximations of the time-harmonic Maxwell equations using the formulation proposed by Kikuchi. The focus is on addressing electromagnetic scattering problems in materials described by Drude-like models. Such materials play a key role in the design of metasurfaces, where precise control of electromagnetic wave interactions is crucial. The ROM significantly accelerates simulations, making it feasible to perform optimization tasks and parametric studies, which are otherwise computationally prohibitive using the high-dimensional model. Therefore, this ROM approach provides a powerful tool for the design of metasurfaces.

References

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