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# A Mixed Tree-Cotree Gauge for Reduced Order Models for the Time-Harmonic Maxwell's Equations

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In this work, we present the application of model order reduction (MOR) approaches to the time-harmonic Maxwell's equations. The resulting reduced order models (ROMs) enable a significantly more efficient evaluation compared to the computationally expensive full-order models. However, as observed in previous studies [1, 2], spurious modes occur when solving the reduced problem, leading to unphysical solutions appearing as added gradient fields.

To address this issue, we propose a mixed tree-cotree gauge to remove the spurious modes from the ROM [3]. This variant is based on the tree-cotree gauge [4] using the fact that with Nédélec edge elements, the degrees of freedom (DoFs) can be associated with the edges of the mesh. The gauge then decomposes the DoFs into two orthogonal subspaces by finding a spanning tree and its cotree on the mesh. While the cotree DoFs can represent the curl-operator, the tree DoFs span the gradient space. By condensing the original problem size to the cotree DoFs, we effectively eliminate the non-trivial gradient space. We then build the ROM based on the full-order model solutions obtained on the cotree DoFs. Thereby, we prevent the emergence of spurious modes in the ROM.

This gauge proves particularly beneficial when approximating a problem with a parameterized domain, as the validity of the gauge relies solely on the mesh topology. Hence, as long as the topology is maintained, i.e., no remeshing is necessary along the parameterization, the gauge remains valid along parameter variations. When discretizing the full-order problem using isogeometric analysis (IGA), large domain deformations can be represented while maintaining a suitable mesh quality without remeshing.

In the full paper, we demonstrate the construction of a ROM for the Maxwell eigenvalue problem in the cavity, where we track the eigenmodes while deforming the cavity shape. In this setting, possibly many evaluations of the eigenvalue problem become necessary, making the process computationally costly. Therefore, we speed up the algorithm by performing the eigenmode tracking on the ROM. We demonstrate the efficient implementation of the mixed tree-cotree gauge, which further allows for a high accuracy of the ROM. Thereby, constructing the ROM and tracking the eigenmodes on this space can outperform one run of the tracking on the full order model.

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