URSI-B EMTS 2025, Bologna, 23–27 June

UAPO Solution for Plane Wave Diffraction by a Resistive Half-Plane Between Isorefractive Media

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Abstract

The plane wave diffraction by a truncated resistive sheet residing at the interface between two lossless isorefractive media is solved. The diffraction contribution is originated by the planar junction incorporating the resistive half-plane and the semi-infinite isorefractive interface that is not occupied. The proposed solution is achieved by using the equivalence principle and by applying the Uniform Asymptotic Physical Optics method to each isorefractive half-space. This requires finding new expressions for the reflection and transmission coefficients related to the resistive half-plane. Numerical tests prove the ability of the UAPO diffracted field to compensate the discontinuities of the Geometrical Optics field at the shadow boundaries.

1. Introduction

The term "isorefractive" was coined by Uslenghi [1] to define the space occupancy by two media having the same wavenumber ($k_1 = k_2 = k$), but different intrinsec impedance, i.e., $\zeta_1 \neq \zeta_2$. This implies that the propagation direction of plane waves crossing the common interface is unaffected by the change of medium. In addition, the reflection and transmission coefficients are independent on the incidence direction.

The boundary condition between two isorefractive media can be obtained by designing artificial media as suggested by Tyo in [2], where a class of artificial materials that are isorefractive with the surrounding free space was proposed. Obviously, this condition changes when a thin layer of different material is located between the isorefractive half-spaces. At first, the presence of a PEC half-plane was considered in [3]. The scattering problem involving such a geometry was also addressed and solved in [4] by exploiting the equivalence principle and employing the Uniform Asymptotic Physical Optics (UAPO) approach to each isorefractive medium for evaluating the diffraction contribution in the framework of the Uniform Geometrical Theory of Diffraction (UTD) [5]. Research articles referring to penetrable half-screens at the interface are also available in literature [6-8], but only two-dimensional scattering problems characterized by incidence direction that is perpendicular to the screen edge were studied.

The following section is devoted to solve the three-dimensional diffraction problem arising from the interaction between propagating plane waves and a truncated thin layer, which is characterized by resistive boundary conditions and located at the interface between two lossless isorefractive media (see Fig. 1). The equivalence principle and the UAPO-based method are applied to this end. Note that the reflection and transmission coefficients used in [9, 10] cannot be accepted and new expressions must be attained to account for the isorefractive media.

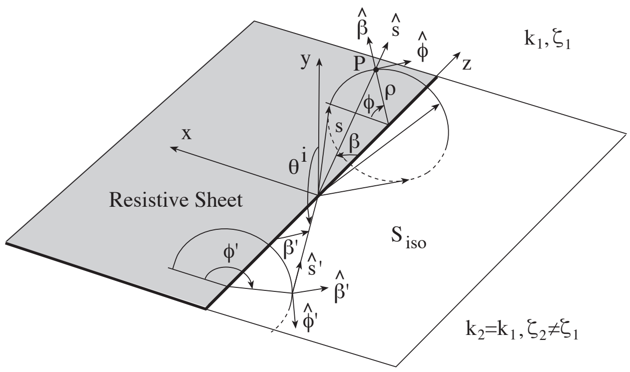

Figure 1. Diffraction problem.

2. UAPO Diffracted Field

The geometry of the diffraction problem is depicted in Fig. 1. The resistive sheet occupies half-interface and its edge coincides with the *z*-axis of the reference system having the *y*-axis pointing toward the medium 1. A plane wave propagates in this medium according to the unit vector $\hat{s}' = -\sin\beta'\cos\phi'\hat{x} - \sin\beta'\sin\phi'\hat{y} + \cos\beta'\hat{z}$, and the incident field is $\underline{E}^i = (E^i_{\beta'}\hat{\beta}' + E^i_{\phi'}\hat{\phi}')\exp(-jk\hat{s}'\cdot\underline{r})$, with $\underline{r}$ denoting the position of the observation point *P*.

The incident field produces reflection and transmission contributions depending on the illuminated surface, thus making jumps in the GO field at the reflection (RSB) and transmission (TSB) boundaries. This suggests to evaluate the diffracted field as originated by the discontinuity of a planar junction. In addition, the presence of two different materials in the upper and lower half-spaces recommends to split the original diffraction problem in two diffraction problems and to employ the equivalence principle for the computation of the UAPO diffracted field. According to the above statements, the UAPO-based method used in [4] can be also applied to the diffraction problem in Fig. 1. This method requires electric and magnetic equivalent surface currents radiating in the half-space 1 (2) to evaluate the UAPO diffracted field $\underline{E}_1^d$ ($\underline{E}_2^d$) in such a region (see Fig. 2). At the end of the procedure, it results:

$$\underline{E}_1^d = \begin{pmatrix} E_{1_\beta}^d \\ E_{1_\phi}^d \end{pmatrix} = \underline{\underline{D}}_1 \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \frac{\exp(-jks)}{\sqrt{s}} = \left[C_{res}^{d+} \underline{\underline{N}}_{res}^{+} + C_{iso}^{d+} \underline{\underline{N}}_{iso}^{+} \right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \frac{\exp(-jks)}{\sqrt{s}} \tag{1}$$

$$\underline{E}_2^d = \begin{pmatrix} E_{2_\beta}^d \\ E_{2_\phi}^d \end{pmatrix} = \underline{\underline{D}}_2 \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \frac{\exp(-jks)}{\sqrt{s}} = \left[C_{res}^{d-} \underline{\underline{N}}_{res}^{-} + C_{iso}^{d-} \underline{\underline{N}}_{iso}^{-} \right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \frac{\exp(-jks)}{\sqrt{s}} \tag{2}$$

wherein s is the distance from the diffraction point to P and

$$C_{res,iso}^{d\pm} = \frac{F\left(2ks\sin^2\beta'\cos^2\left(\frac{\phi_{res,iso} \pm \phi'_{res,iso}}{2}\right)\right)}{\sin^2\beta'\left[\cos\phi_{res,iso} + \cos\phi'_{res,iso}\right]} \times \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k}} \tag{3}$$

The function F is the UTD transition function [5], and the angles $\phi'_{res,iso}$ define the incidence direction with respect to the resistive sheet and the isorefractive interface, respectively, whereas $\phi_{res,iso}$ are associated to P. The matrices $\underline{\underline{N}}_{iso}^{\pm}$ are the same as in [4], and

$$\underline{\underline{N}}_{res}^{\pm} = \underline{\underline{N}}_1 \underline{\underline{N}}_2 \underline{\underline{N}}_3 \underline{\underline{N}}_4^{\pm} \underline{\underline{N}}_5 \tag{4}$$

with

$$\underline{\underline{N}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \tag{5}$$

$$\underline{\underline{N}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \tag{6}$$

$$\underline{\underline{N}}_3 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \tag{7}$$

$$\underline{\underline{N}}_4^{+} = \begin{pmatrix} 0 & (1-R_{res\perp})\sin\beta'\sin\phi' \\ 1+R_{res\parallel} & 0 \end{pmatrix} \tag{8a}$$

$$\underline{\underline{N}}_4^{-} = \begin{pmatrix} 0 & -T_{res\perp}\sin\beta'\sin\phi' \\ -T_{res\parallel} & 0 \end{pmatrix} \tag{8b}$$

$$\underline{\underline{N}}_5 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \tag{9}$$

wherein $R_{res\parallel}, T_{res\parallel}, R_{res\perp}, T_{res\perp}$ are the new reflection (R) and transmission (T) coefficients for the parallel ($\parallel$) and perpendicular ($\perp$) field components to the ordinary plane of incidence when considering the resistive half-sheet between two isorefractive media.

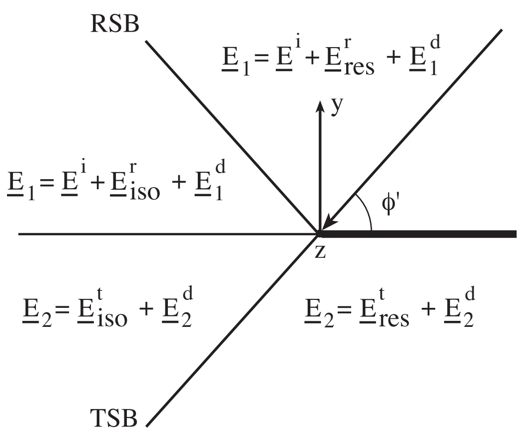

Figure 2. Total field contributions.

3. Numerical Examples

The plots in Figs. 3 and 4 demonstrate the ability of the UAPO diffracted field to compensate the jumps of the GO field so producing a continuous total field. They refer to an incidence direction $\beta' = 70°, \phi' = 60°$ when P moves on a circular path with radius $\rho = 6\lambda$, where λ is the wavelength.

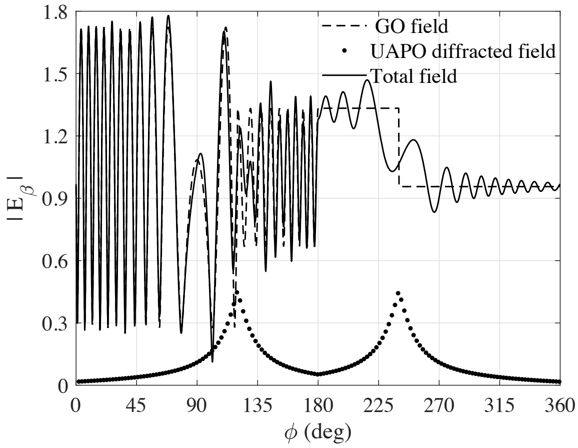

Figure 3. Field β-component when $E^i_{\beta'} = 1, E^i_{\phi'} = 0$.

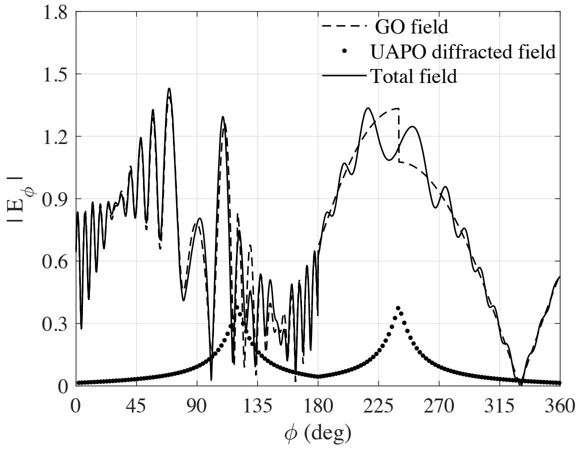

Figure 4. Field ϕ-component when $E^i_{\beta'} = 0, E^i_{\phi'} = 1$.

The isorefractive media are characterized by $\zeta_1 = \zeta_0 / \sqrt{2}$, $\zeta_2 = \zeta_0 \sqrt{2}$, and $k = k_0 \sqrt{2}$, where ζ_0 and k_0 are the intrinsec impedance and the wavenumber of free space, respectively. The resistive half-sheet has thickness $d = 0.02\lambda$, relative permittivity $\varepsilon_r = 8 - j0.1$ and free-space permeability.

4. Conclusions

The plane wave diffraction by a resistive half-sheet located at the interface between two isorefractive materials has been addressed and solved. In particular, a UAPO-based method has been applied to evaluate the diffracted field in accordance with the equivalence principle. The resulting total field has been proved to be continuous crossing the reflection and transmission boundaries.

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