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Electrodynamics of Temporal Interfaces: Analysis and design of Time-switched metamaterial devices

Davide Ramaccia(1)*, Luca Stefanini(1)(2), Andrea Alù(2), Alessandro Toscano(1), and Filiberto Bilotti(1)
(1) RomaTre university, Rome, Italy (*davide.ramaccia@uniroma3.it)
(2) Advanced Science Research Center, CUNY, New York City, NY (USA).

Abstract

Time-varying metamaterials, engineered materials with dynamically adjustable properties, exhibit particular interest when modulated in a step-like fashion, enabling the creation of temporal interfaces. Our prior research investigated scattering phenomena within these metastructures, encompassing configurations with single, double, and multiple temporal interfaces, both in unbounded and guided wave environments. This work focuses on the analytical techniques employed for modeling and designing such systems, with particular emphasis on the temporal transfer matrix, mode-matching, and Laplace transform methods.

1. Introduction

Recent advancements in metamaterials and metasurfaces have enabled unprecedented control over wave propagation through the introduction of time-varying properties [1]-[3]. This dynamic approach unlocks new functionalities, such as manipulating the temporal characteristics of electromagnetic (EM) waves and circumventing limitations imposed by passivity. Notable examples include the realization of matching conditions surpassing the Bode-Fano limit [4], temporal Fabry-Perot resonators [5], wave aiming [6], effective temporal media [7], broadband absorbers [8], lasing media [9], temporal metasurfaces [10]-[11], and efficient phase conjugation [12]. These demonstrations typically rely on abrupt changes in material properties across the entire propagation domain, presenting significant challenges for practical implementation. Several approaches have been proposed to mitigate this constraint. These include investigations into propagation along time-switched transmission lines [13] and within optical waveguides [14]-[16]. Another strategy leverages the effective medium properties experienced by guided waves. By manipulating the boundary conditions of guiding structures, it becomes possible to induce temporal interfaces exhibiting analogous scattering phenomena. Recent studies have explored three distinct anomalous scattering processes within this framework: i) the generation of forward and backward propagating modes, each experiencing a unique change in material properties [17], ii) the diffusion of a continuous spatio-temporal spectrum of plane waves [18], and iii) the radiation of a directive beam in the end-fire direction at a frequency significantly higher than the source frequency [19].

This contribution focuses on the analytical methods and design approaches employed for modeling and designing time-modulated metastructures characterized by abrupt changes in bulk or effective material properties. We provide a concise overview of each analytical method and its specific application within the context of the corresponding structure. Figure 1 summarizes the time-modulated metastructures that will be presented, categorized based on the analytical method used to determine the characteristics of the scattered waves from the time interface.

2. Analysis of Time-modulated metastructures

2.1 Temporal transfer matrix method

The transfer matrix method (*TMm*) is a well-established technique for analyzing the propagation and scattering of electromagnetic (EM) waves through layered media. By leveraging the principle of space-time duality, a temporal analog of the *TMm* has been developed, enabling the analysis and design of temporal metastructures composed of a sequence of temporal interfaces.
As derived in [5], [9], the relationship between the original and forward/backward scattered fields can be written as:

$$\begin{bmatrix} E_b^+ \\ E_b^- \end{bmatrix} = \left(MM_1 \prod_{k=1}^{K} DM_k MM_{k+1} \right) \begin{bmatrix} E_a^+ \\ 0 \end{bmatrix} \qquad (1)$$

where E_a^+ is the original field in the initial medium and E_b^+, E_b^- are the forward and backward propagating fields in the final one. In (1), MM_k and DM_k identify the 2x2 matching and delay matrices of each slab composing the multilayered structure, respectively. Such method has been successfully used for analyzing the reflection and transmission from a temporal slab [5] and a multilayered structure [9]. This method allows to derive the scattering response in terms of the new parameter effective application time, that plays the same role of the electric thickness for static spatial EM structures. As reported in Figure 1, the analytically derived forward and backward scattering coefficients agree very well with the full wave numerical results obtained by using a Finite Difference Time Domain EM simulation. Exploiting the effectiveness and reliability of the proposed *TTm*, some novel devices based on temporal multilayered slabs have been proposed, such as temporal Fabry-Perot resonators, transparent multilayered structures and lasing media [5],[9].

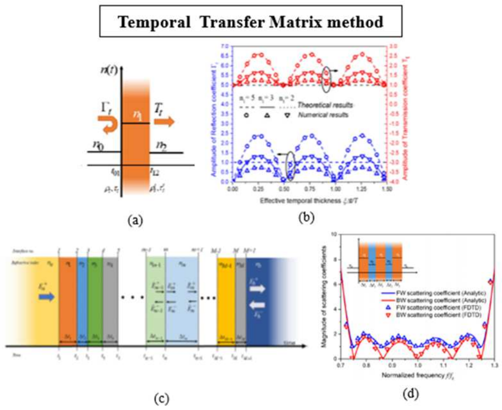

Figure 1. Temporal multilayered structures and corresponding results obtained by using the temporal transfer matrix method.

2.2 Mode-matching method

The mode-matching method (*MMm*) is a versatile technique employed in the formulation of boundary-value problems. It relies on matching field components across spatial boundaries between different regions, making it particularly well-suited for analyzing boundary-value problems. *MMm* has been extensively applied in the context of scattering and transmission problems, as well as in the analysis of transmission lines, particularly in cases involving discontinuities within waveguides. The method has been applied to derive the amplitude of the scattered waves at a boundary-induced time interface within a parallel-plate waveguide as illustrated in Figure 2. Specifically, considering TE-mode propagation within the waveguide, we can impose the continuity on the induction fields **B** and **D** at the time interface as follows:

$$\begin{cases} B_z^{t_0^+}(y,z) = \sum_{m=1}^{+\infty}\left(a_m e^{i\omega_m t} - b_m e^{-i\omega_m t}\right) B_{z,m}^{t_0^+}(y,z) \\ B_y^{t_0^+}(y,z) = -\sum_{m=1}^{+\infty}\left(a_m e^{i\omega_m t} - b_m e^{-i\omega_m t}\right) B_{y,m}^{t_0^+}(y,z) \\ D_x^{t_0^+}(y,z) = -\sum_{m=1}^{+\infty}\left(a_m e^{i\omega_m t} + b_m e^{-i\omega_m t}\right) D_{x,m}^{t_0^+}(y,z) \end{cases}, \quad (2)$$

where the fields after the time interface have been written as a superposition of a theoretically infinite set of forward and backward modes, each of which characterized by a specific amplitude a_m, b_m and freqeuncy $\pm\omega_m$. Equation (2) has been used for analysing two different time-switched structures: the first one considers the case of sudden variation of the distance between the two plates as in [17], whereas the second one considers the case of sudden disappearance of the top plate as in [18]. The method allows deriving the amplitude of the scattered modes excited in the new waveguide for different distances of the plates (Figure 2(b)-(c)) and the energy distribution of the temporal rainbow emerging from the time-switched metastructure (Figure 2(e)).

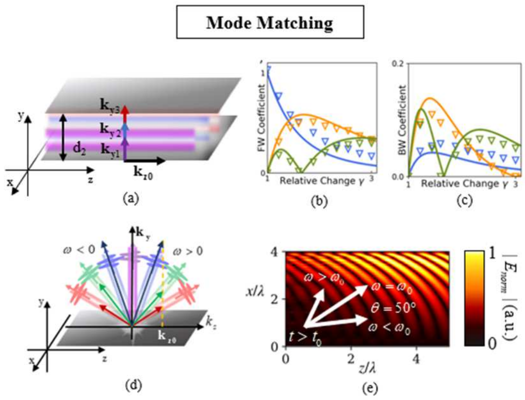

Figure 2. Parallel-plate waveguide platforms for achieving boundary-induced time interfaces and corresponding results obtained by using mode matching method.

2.2 Laplace method

The Laplace method (*Lm*) utilizes the Laplace transform of the wave equation to obtain concise and physically insightful solutions for electromagnetic (EM) fields emanating from time-modulated metastructures. By employing the Laplace transform, initial value problems can be readily incorporated, and the formally exact solution can be derived in the form of an inverse Laplace transform integral. As illustrated in Figure 3, this method has been effectively applied to determine the scattered field from the metasurface and its corresponding radiation pattern. The *Lm* enables a direct connection between the initial surface wave confined along the metasurface, arising from the finite capacitive value of the surface impedance, and the radiated set of plane waves that emerge following the switching time

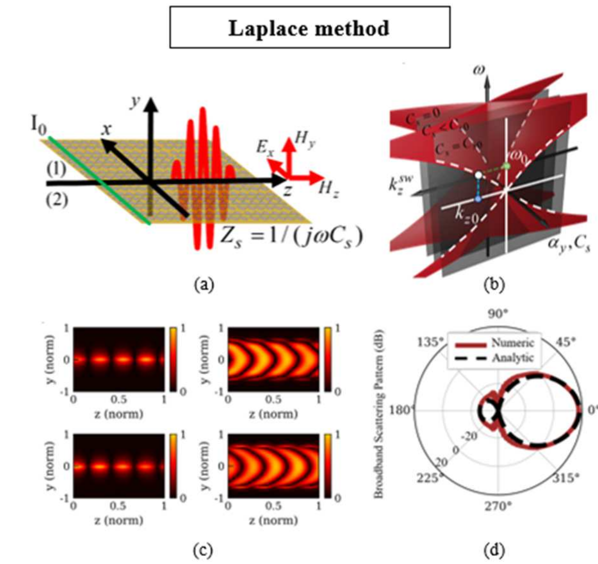

Figure 3. Time-switched metasurface and corresponding results obtained by solving the wave equation in the Laplace domain with proper initial values.

3. Conclusions

This work has presented a comprehensive overview of the analytical methods employed for modeling and designing time-modulated metastructures, focusing specifically on three key approaches: the temporal transfer matrix method, coupled-mode theory, and the Laplace transform method. The temporal transfer matrix method derives from its spatial counterpart, providing a robust framework for analyzing wave propagation in cascaded temporal interfaces. Mode-matching method allows understanding the interaction between different modes supported by a guiding system before and after the temporal switching. Finally, the Laplace transform has been used for solving the time-domain wave equation in the presence of time-dependent material properties.

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