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Synthesis of Spatially Distributed Caustics for Curved Wave Beam Radiation in the Near Field

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Abstract

In this contribution, we present a method for synthesizing focused beams by shaping ray caustics at microwave and millimeter-wave frequencies. Caustics, as described in geometrical optics, constitute high-intensity field curves on which optical rays converge. They can be effectively adopted to focus beams in near field along specific spatial trajectories. The proposed technique offers a physics-based approach to introduce additional free parameters to synthesize bendable focused beams, surpassing more conventional methods for generating non-diffractive beams. This method has promising applications in advanced communication systems and in innovative near-field launcher design.

1 Introduction

Caustics theory finds applications in engineering and applied physics, including optics, near-field focusing, imaging, and gravitational lensing. In optics, caustics are high-intensity light trajectories arising when light interacts with curved surfaces. They can take the form of focal curves or intricate light patterns produced by glass surfaces. Under the ray-optical framework, caustics can be defined as the envelope of tangents to light rays, on which rays converge and focus in a region of high field intensity. Depending on their geometric and topological properties, caustics have been classified in seven fundamental types, including Airy, Pearcey, and Swallowtail caustics [1].

Beyond theoretical interest, ray caustics exhibit remarkable applications in optical communications [1, 2] and in high-resolution imaging [3, 4]. Indeed, an appropriate design of ray-caustic position and shape allows bending the radiated beam in the near field region along specific trajectories. However, a key challenge in these areas is the ability to engineer caustics in a simple but effective way (i.e., with controllable trajectories and intensity distributions), especially at microwaves and millimeter waves, in which not so much attention has been paid up to now.

In this framework, it is quite interesting to consider a class of exact solutions of Helmholtz wave equation, namely the Bessel beams [5, 6], in a new light. Indeed, they can be interpreted as special cases of ray-caustic shaping theory, since the Bessel main beam can be synthesized as a straight locus of caustics. This property makes Bessel beams useful in applications requiring precise control of beam shape and intensity distributions, such as in optics [7, 8] and at microwaves/millimeter waves [9, 10, 11], by considering both free-space and bounded domains [12] as surrounding media for radiation.

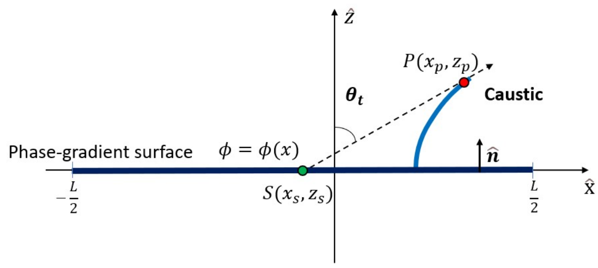


Figure 1. Near-field synthesis based on ray-caustic shaping. A phase-gradient surface is designed to radiate a prescribed curved beam.

To develop caustic engineering, it is required to take advantage of additional parameters when modeling beam properties, leveraging concepts from Geometrical Optics (GO). Indeed, according to GO, the envelope of the tangents to reflected or refracted rays corresponds to regions of high beam intensity, thus offering an effective way to concentrate radiated energy along curved trajectories, beyond straight lines. This contribution explores an innovative near-field synthesis technique based on ray-caustic shaping, offering a physics-based approach that surpasses more traditional non-diffractive beams.

The paper is organized as follows. Section II reviews the theoretical fundamentals of ray-caustic shaping in a 2-D equivalent environment, which has already been proven to be effective in [13, 14, 15]. Section III presents numerical examples demonstrating the generation of distributed caustics in the near field, with the purpose to radiate curved beams. Finally, conclusions are drawn.

2 Theoretical model

Let us suppose an impenetrable phase-gradient surface placed along the x-axis, able to synthesize a prescribed phase distribution $\phi = \phi(x)$, as shown in Fig. 1. For the sake of simplicity and to show the effectiveness of the proposed method, we restrict ourselves to a 2-D equivalent environment. The trajectory described by the target ray caustic is defined by the curve $z = f(x)$. The objective of this study is to determine the spatial phase profile $\phi(x)$ at the plane $z = 0$, which enables the synthesis of the desired curved caustic. A caustic can be defined as the envelope of ray tangents, where each point x on the plane $z = 0$ corresponds to a point on the caustic. This relation is established through a general tangent with slope $\theta_t(x)$, related to the caustic curve by the expression $\tan\theta_t(x) = \frac{df(x)}{dx}$.

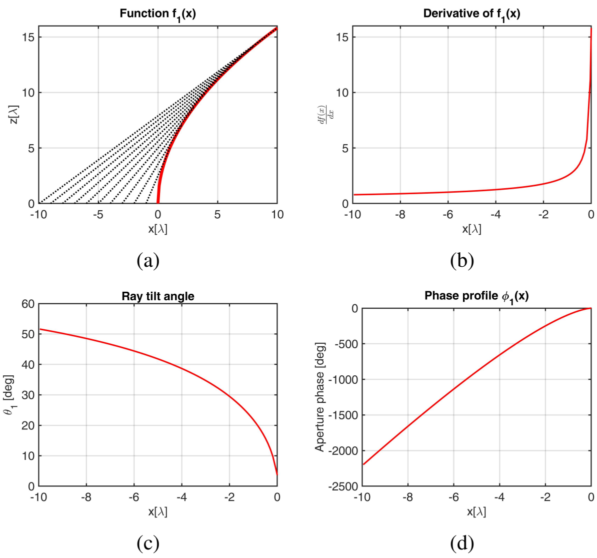


Figure 2. (**a**) Plot of the function $f_1(x)$, which defines the desired caustic. (**b**) Representation of the first derivative of $f_1(x)$. (**c**) Illustration of the ray tilt angle $\theta_1(x)$ along the propagation path. (**d**) Representation of the phase profile $\phi_1(x)$, which generates the desired caustic.

Consequently, the spatial derivative of the phase function can be expressed as

$$\frac{d\phi(x)}{dx} = \frac{2k\tan\theta_t(x)}{1+\tan^2\theta_t(x)}, \tag{1}$$

being k the free-space wavenumber. By simply recalling, by geometrical considerations, that $\pi - \theta_t(x) = \arctan\frac{df(x)}{dx}$, the phase function $\phi(x)$ can be finally derived. Once the target phase distribution on the aperture and the position of the radiators are known, either analytically or numerically, the target current density distribution able to radiate the prescribed caustic $\mathbf{J}(\mathbf{x}) = e^{-j\phi(x)}\hat{\mathbf{y}}$ is defined. The 2-D radiated electric field can be then expressed as

$$\mathbf{E}(\mathbf{r}) = -\hat{\mathbf{y}}\, jk\mathbf{J}(\mathbf{r})G_{2D}(R) \tag{2}$$

where $G_{2D}(R)$ is the 2-D scalar Green's function for an unbounded medium, and R is the distance between source and observation points. By imposing the correct phase function $\phi(x)$, the electric field is focused along the desired caustic trajectory, thus concentrating the radiated energy at the target locations.

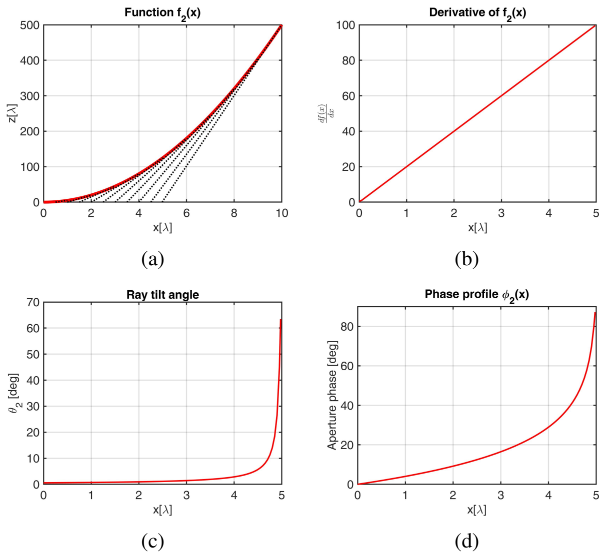


Figure 3. (a) Plot of the function $f_2(x)$, which defines the desired caustic. (b) Representation of the first derivative of $f_2(x)$. (c) Illustration of the ray tilt angle $\theta_2(x)$ along the propagation path. (d) Phase profile $\phi_2(x)$, which generates the desired caustic.

3 Numerical Results

As numerical examples, we consider two target caustics defined by the functions $f_1(x) = \alpha\sqrt{x}$, and $f_2(x) = \alpha x^2$, with $\alpha = 5$, both extending over a prescribed length L at $z = 0$. In Figs. 2-3 the desired caustic functions (and the associated ray tangents), the function derivatives, the associated ray tilts with respect to the z-axis, and the final phase profiles are shown for clarity. The goal is to demonstrate that the radiated field lays along the trajectories $f_{1,2}(x)$.

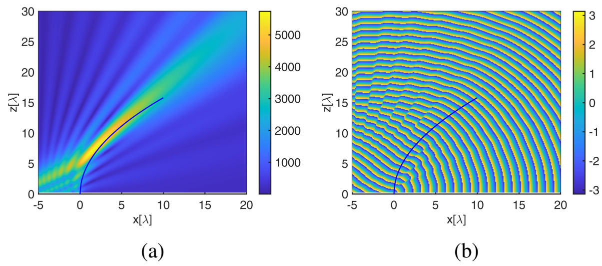
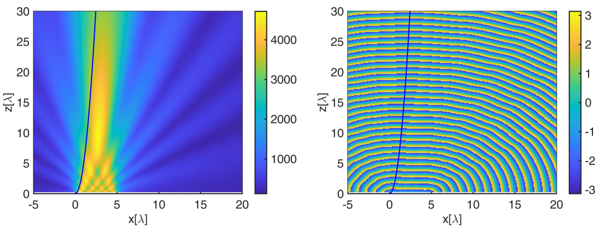


Figure 4. (**a**) Amplitude and (**b**) Phase of the radiated field generated by the phase profile $\phi_1(x)$. The results demonstrate the alignment of the radiated field along $f_1(x)$.

The numerical results, shown in Fig. 4 and Fig. 5, validate the proposed theoretical model by confirming that the field aligns with the desired caustic paths. From the phase plots in Figs. 4(b)-5(b), it is worth noting that the local phase profiles in the neighborhood of the desired caustics are locally planar and always orthogonal to the caustic lines, as required from GO considerations. The proposed results pave the way to new applications of ray caustics to shape focused beams along arbitrary trajectories, thus enabling innovative opportunities for high-speed near-field communications.



(a) (b)

Figure 5. (**a**) Amplitude and (**b**) Phase of the radiated field generated by the phase profile $\phi_2(x)$. The results demonstrate the alignment of the radiated field along $f_2(x)$.

4 Conclusion

We introduced a novel method for near-field beam shaping through caustic synthesis. Caustics are defined as the envelope of tangents to reflected or refracted rays, in which high field intensity naturally occurs. This feature makes caustics particularly effective for focusing beams along curved spatial trajectories. The proposed approach provides additional parameters for the design of innovative near-field launchers and generalizes more conventional techniques based on nondiffractive beams.

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