Characteristic Modes of PEMC Objects

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Characteristic mode (CM) analysis of lossless conducting and dielectric objects is today a well known tool in antenna and scattering theory. One of the most recent additions to this topic is the formulation of scattering-based characteristic modes of lossless nonreciprocal systems [1]. The purpose of this presentation is to discuss new integral-operator-based CM formulations for nonreciprocal objects with perfect electromagnetic conductor (PEMC) boundary condition.

The PEMC boundary condition [2] is lossless and isotropic. It is defined using the real normalized PEMC admittance Y as $\mathbf{n}\times(\eta\mathbf{H}+Y\mathbf{E})=\mathbf{0}$, where $\mathbf{n}$ is the normal of the boundary and η the intrinsic impedance of the surrounding lossless space. The PEMC boundary is nonreciprocal, except for the limiting special cases $Y\to\pm\infty$ (perfect electric conductor, PEC) and $Y=0$ (perfect magnetic conductor, PMC).

The new CM formulation is based on using the PEMC boundary condition to either eliminate the magnetic surface current $\mathbf{M}$ from the electric field integral equation (EFIE) or the electric surface current $\mathbf{J}$ from the magnetic field integral equation (MFIE). After that, the final important step is to select the correct weight operator following [3], and the resulting generalized eigenvalue equations are

$$\left(\mathcal{T}-\tfrac{1}{Y}\mathcal{K}^{+}\right)[\mathbf{J}_n]=(1-i\lambda_n)\left(\mathrm{Re}\{\mathcal{T}\}-\tfrac{i}{Y}\,\mathrm{Im}\{\mathcal{K}\}\right)[\mathbf{J}_n], \tag{1}$$

$$\left(\mathcal{T}+Y\mathcal{K}^{+}\right)[\mathbf{M}_n]=(1-i\lambda_n)\left(\mathrm{Re}\{\mathcal{T}\}+iY\,\mathrm{Im}\{\mathcal{K}\}\right)[\mathbf{M}_n], \tag{2}$$

where $\mathcal{T}$, $\mathcal{K}$, and $\mathcal{K}^{+}$ are the usual tangential surface integral operators. Solving (1) or (2) and using the PEMC boundary condition $\eta\mathbf{J}_n=Y\mathbf{M}_n$ gives a set of characteristic eigenvalues λ_n and the corresponding eigencurrents $\mathbf{J}_n,\mathbf{M}_n$. Both formulations give the same results, but for numerical stability the EFIE-based formulation (1) is preferred if $|Y|\gg 1$ and the MFIE-based formulation (2) is preferred if $|Y|\ll 1$.

Although not immediately obvious from the above formulations, the characteristic eigenvalues λ_n are the same as for a PEC object with the same geometry. That is, the eigenvalues λ_n are independent of the PEMC admittance Y, while the corresponding eigencurrents $\mathbf{J}_n,\mathbf{M}_n$ depend on Y. Numerical results to verify this will be presented at the conference.

References

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