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Time-Domain Physical Bounds on the Causal Response of Electromagnetic Systems: A Brief Summary of Recent Developments

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Abstract

Early- and late-time bounds on the causal response of linear time-invariant systems are discussed. Their applications to selected electromagnetic (EM) systems are briefly reviewed. Furthermore, a novel closed-form expression is presented for the bound on the space-time voltage distribution induced along a voltage-source excited transmission line (TL) loaded by a serial *RC* load.

1 Introduction

Time-varying (4-D) electromagnetic (EM) metasurfaces are envisaged as a promising means for the deliberate control of the radio environment in future communication systems. Their engineering design, however, is currently hindered by the lack of analytical studies and computational tools that were traditionally developed under the assumption of time invariance [1]. This issues has recently initiated research on EM radiation and scattering phenomena by *time-varying* (TV) EM structures (e.g., [2, 3, 4, 5, 6, 7]). A contribution to these efforts is the development of time-domain (TD) physical bounds [8, 9, 10, 11] that enable a straightforward benchmarking of TV systems and devices directly in the original (space-time) domain. Selected examples of their use are presented in this paper.

2 Integral Representations of the Causal Response and its Bounds

The (real-)frequency-domain (FD) response of a linear time-invariant system is restricted due to the analyticity of its continuation in the right-half of the (complex-)frequency plane [12, Sec. XIII]. This behavior necessarily impose restrictions on the corresponding TD response. To derive such limitations, the causal TD response of a linear time-invariant system is expressed as [8, Eq. (6)]

$$\Gamma(t) = \gamma_\infty \delta(t) + \gamma_0 \mathrm{H}(t) + 2\sum_{n=1}^{N} \gamma_n \cos(\omega_n t)\mathrm{H}(t) + \Omega(t). \tag{1}$$

where γ_∞, γ_0, γ_n and ω_n are real-valued coefficients, $\Omega(t)$ represents the causal TD remainder function, $\mathrm{H}(t)$ denotes the Heaviside unit-step function and $\delta(t)$ is the Dirac-delta distribution. Since the one-sided Laplace transform of $\Omega(t)$, denoted as $\hat{\Omega}(s)$ with $s \in \mathbb{C}$ being the transform parameter (= complex frequency), is an analytic function in the right-half of the complex-frequency plane and along its boundary and $\hat{\Omega}(s) = o(1)$ as $s \to \infty$, Cauchy's theorem yields its integral representations that can be transformed to TD as follows

$$\Omega(t) = \frac{2\mathrm{H}(t)}{\pi} \int_{\omega=0}^{\infty} \mathrm{Re}\left[\hat{\Omega}(\mathrm{i}\omega)\right] \cos(\omega t) \mathrm{d}\omega, \tag{2}$$

and

$$\partial_t^{-1}\Omega(t) = -\frac{2\mathrm{H}(t)}{\pi} \int_{\omega=0}^{\infty} \mathrm{Im}\left[\hat{\Omega}(\mathrm{i}\omega)\right] \frac{1-\cos(\omega t)}{\omega} \mathrm{d}\omega, \tag{3}$$

where ∂_t^{-1} denotes the time-integration operator. Representations (2) and (3) can be used to derive early- and late-time bounds on the causal TD response, respectively. Likewise the FD bounds [13], the TD bounds are determined by the asymptotic behavior of $\hat{\Omega}(s)$. Indeed, if $\mathrm{Re}\left[\hat{\Omega}(\mathrm{i}\omega)\right]$ is either positive or negative for all $\omega \in \mathbb{R}$, which applies to the examples presented in [8, 9, 10], we may write (cf., [8, (11) and (12)])

$$|\Omega(t)| \leq \lim_{s\to\infty} |s\hat{\Omega}(s)| \mathrm{H}(t). \tag{4}$$

In a similar manner, assuming that $\mathrm{Im}\left[\hat{\Omega}(\mathrm{i}\omega)\right]$ is either positive or negative, we get [8, (15) and (16)]

$$|\partial_t^{-1}\Omega(t)| \leq 2\hat{\Omega}(0)\mathrm{H}(t). \tag{5}$$

In a number of applications, however, the definite sign requirement does not hold true. In such cases, one can proceed as described in [11], where (5) has been derived under more general assumptions. The corresponding early-time bound (with the additional factor of 2) can also be established for a larger class of systems. Selected examples of applications are further presented.

3 Illustrative Applications

The TD bounds (4) and (5) have been previously applied to a number of TD EM problems:

- TD bounds on the pulse reflection at passive *RLC* loads and high-contrast thin sheets are analyzed in [9];
- The performance of a matching circuit in a bounded time window $\mathscr{W} = \{0 \le t \le T, T > 0\}$, is studied in [8] using the following coefficient

$$\eta = \frac{\int_{\tau=0}^{T} [v_{\mathrm{i}}(\tau) - v_{\mathrm{r}}(\tau)]^2 \mathrm{d}\tau}{\int_{\tau=0}^{T} v_{\mathrm{i}}^2(\tau) \mathrm{d}\tau}, \tag{6}$$

  where v_{i} and v_{r} represent the incident and reflected pulses, respectively. Its TD bound can be readily derived using (4);
- The theory of Herglotz functions is employed in [10] to derive physical bounds on the TD response of passive systems. Their applications to the TD response of a dielectric material are presented;
- The worst-case shielding efficiency of a planar conductive slab is analyzed in [14] using the TD parameter defined as

$$\bar{S} = -20 \log_{10}[\max[\partial_t^{-1} T(t)]], \tag{7}$$

  where $T(t)$ is the pertaining TD transmission coefficient. Its TD bound has been derived via (5) and the high-contrast, thin-sheet approximation [15, Sec. 4.1];
- With the aid of the (generalized) late-time bound (5), the maximum TD voltage induced along a loaded transmission line (TL) is analyzed in [11]. Closed-form expressions for TD bounds are in [11] derived for both purely resistive and serial *RL* loads.

Following the lines of reasoning [11], we may derive a closed-form TD bound on the space-time voltage distribution along a TL that is loaded by a serial *RC* load, i.e.,

$$\begin{aligned} |V(x,t)| \le & \tfrac{1}{2} V_{\mathrm{m}} (1 - \Gamma_1)(1 + |\Gamma_2^{\infty}|) \\ & + V_{\mathrm{m}} [|\Gamma_1| + |1 - \Gamma_2^{\infty}(1 - \Gamma_1)|], \end{aligned} \tag{8}$$

with

$$\Gamma_1 = \frac{R_{\mathrm{S}} - Z_{\mathrm{C}}}{R_{\mathrm{S}} + Z_{\mathrm{C}}} \text{ and } \Gamma_2^{\infty} = \lim_{s \to \infty} \hat{\Gamma}_2(s) = \frac{R_{\mathrm{L}} - Z_{\mathrm{C}}}{R_{\mathrm{L}} + Z_{\mathrm{C}}}, \tag{9}$$

where V_{m} is the amplitude of the excitation voltage pulse, R_{S} is the input resistance of the voltage source, Z_{C} denotes the characteristic impedance of the TL and R_{L} is the resistance of the *RC* load. Note that the bound (8) is independent of the load capacitance C_{L}.

As an example, we analyze the voltage along a bounded TL that extends over $\{0 \le x \le \ell\}$, where $\ell = 0.10\,\mathrm{m}$ denotes its length. The TL is at its near-end, $x = 0$, excited by a symmetrical trapezoidal pulse of unit amplitude, $V_{\mathrm{m}} = 1.0\,\mathrm{V}$. Its pulse time width, t_{w}, is a quarter of the pulse travel time along the TL, i.e., $c_0 t_{\mathrm{w}} = \ell/4$, and its pulse rise and fall times are a fifth of the pulse time width, i.e., $t_{\mathrm{r}} = t_{\mathrm{f}} = t_{\mathrm{w}}/5$. Fig. 1 shows the excitation pulse shape in the chosen time window of observation $\{0 \le t \le 6\ell/c_0\}$. The voltage pulses at four observation points along the TL, $\{x = n\ell/3, n = 0, 1, 2, 3\}$, are shown in Fig. 2 for $R_{\mathrm{S}} = 10\,\Omega$, $Z_{\mathrm{C}} = 50\,\Omega$, $R_{\mathrm{L}} = 55\,\Omega$ and $C_{\mathrm{L}} = 0.10\,\mathrm{pF}$. As can be seen in Fig. 2, the bound represented by inequality (8) is met.

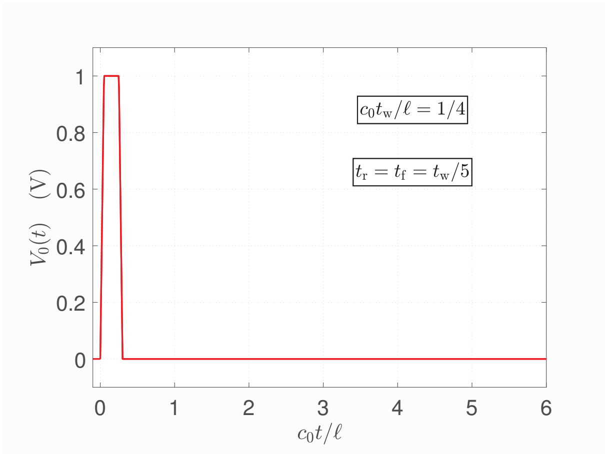

Figure 1. The trapezoidal excitation voltage pulse.

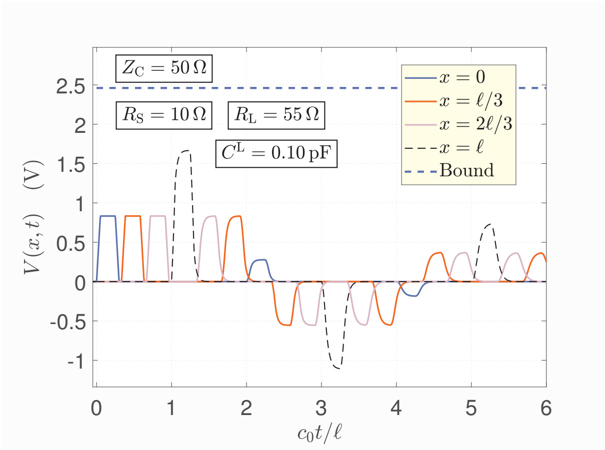

Figure 2. The voltage pulses as observed at $x = \{0, 1/3, 2/3, 1\}\ell$ and the corresponding bound.

4 Conclusions

TD physical bounds on the causal response of a linear time-invariant system and their applications have been briefly discussed. A TD bound on the space-time voltage distribution as induced along a pulsed voltage-excited, finite TL with a serial *RC* load termination has been derived. Illustrative numerical examples have been presented.

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