Influence of Gravitational and Non-Gravitational Forces on Interplanetary Dust

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Abstract

Interplanetary Dust Particles pervade our Solar System, originating from diverse sources such as the Asteroid Belt, Kuiper Belt, and comets that leave dust due to sublimation of ice in the vicinity of Sun. The dynamics of these particles are influenced by a complex interplay of Gravitational and Non-Gravitational forces. The non-Gravitational forces, which include Solar Radiation force, Solar Wind force, and Poynting-Robertson drag, Lorentz force etc. can perturb the orbits of these particles, gradually altering their orbital elements over geological time scales. This paper aims to analyze the perturbations in the semi-major axis and eccentricity of IDPs as a function of their heliocentric distances from the Sun focusing in the inner solar system, by utilizing perturbation equations derived from the fundamentals of celestial mechanics.

1. Introduction

Dust is a major constituent of Solar system objects, which is found on every planet and also, in the interplanetary space. A huge number of Interplanetary Dust Particles (IDPs) are present in the Asteroid belt and Kuiper belt. Occasionally, comets coming from outside of the Solar system contribute dust particles to our Solar system, along with water. These dust particles play a vital role in various observed phenomena such as Zodiacal Light and Meteor Showers (Brownlee et al., 1997) [1]. Observations of comet Hale-Bopp and in-situ dust experiments on Ulysses/Galileo have provided large data sets for IDP science. The Galileo spacecraft made two passages in the Gossamer rings of Jupiter and provided the first in-situ measurement of dusty rings. Our knowledge about the dust surrounding the globe is increased from ground-based observations of meteor streams and Earth orbiting satellites. Though, theoretical modeling, empirical derivations and laboratory studies are possible after successful space observations of IDPs, there has been a lot more to explore for enhancing the present understanding and many new research areas emerge.

2. Solar Radiation Force

The force on a perfectly absorbing particle due to solar (photon) radiation can be viewed as composed of two parts: (a) a radiation pressure term, which is due to the initial interception by the particle of the incident momentum in the beam, and (b) a mass-loading drag, which is due to the effective rate of mass loss from the moving particle as it continuously reradiates the incident energy. The total amount of energy intercepted per second from the radiation beam of integrated flux density S (ergs cm^{-2} sec^{-1}) by a stationary, perfectly absorbing particle of geometrical cross section A is SA. (Burns et al., 2014) [2]. If the particle is moving relative to the Sun with some velocity, the radiation force is given by

$$\boldsymbol{F}_{rad} = \frac{S}{c}\left[\left(1-\frac{\dot{r}}{c}\right) A Q_{PR}\hat{\boldsymbol{p}}\right] \qquad (1)$$

where $\dot{r} = v \cdot \hat{\mathbf{S}}$ is the radial velocity S is a unit vector in the direction of the incident beam, $\hat{p} = \frac{\boldsymbol{c}-\boldsymbol{v}}{c}$ and c is the speed of light; Q_{PR} is the scattering coefficient factor, the factor in parentheses is due to the Doppler effect, which alters the incident energy flux by shifting the received wavelengths. This is the *Radiation Pressure Force* [3].

3. Poynting Robertson Force

Since the particle is losing momentum while its mass is in fact always conserved, the particle is decelerated: it suffers a dynamical drag. The momentum loss per unit time represents, according to the impulse-momentum theorem, a force on the particle of the same amount; this is sometimes called the Poynting-Robertson drag. The net force on the particle is then

$$\boldsymbol{F}_{PR} = \frac{SA}{c}\left[\left(1-\frac{\dot{r}}{c}\right)\hat{\boldsymbol{S}} - \frac{V}{c}\right] \qquad (2)$$

in terms of order v/c. This is the Robertson's Formula.

4. Solar Wind Force

The solar wind force occurs due to the momentum transfer of these particles to the interplanetary dust and is given by [4][5],

$$\boldsymbol{F}_{SW} = \sum_j \frac{\eta_j u^2}{2} A C_{D,j} \hat{\boldsymbol{u}} \tag{3}$$

where u is the orbital velocity of the dust particle and the spatial mass density of the component j of the solar wind flow is given by $\eta_j = n_j m_j$ for mass m_j and number density n_j; A is the geometrical cross section of the grain; $C_{D,j}$ is the dimensionless drag coefficient due to the j-component of the wind flow [6].

5. Dust Dynamics

The above-mentioned forces alter the particle drift towards the Sun and act differently, depending on the dust origin and size. They change the orbital elements (a, e, i, ω, Ω, f) over geologic time scales and can be defined by perturbation equations.

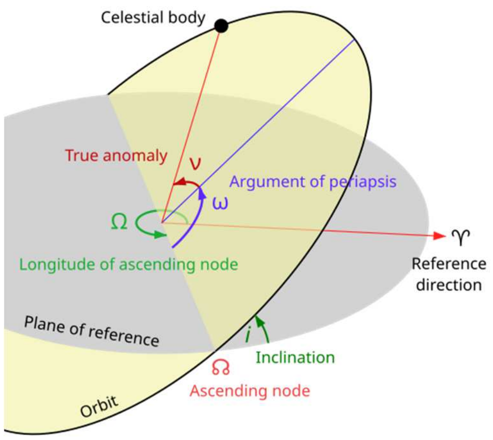

Figure 1. Image showing the Orbital Elements; where the orbital plane (yellow) intersects a reference plane (gray) [7]

5.1 Unperturbed Orbit

We consider a particle of mass m moving in the gravitational field of a fixed-point mass M. Newton's equation of Motion is given by

$$\boldsymbol{F}_g = \frac{-\mu m}{r^3} \boldsymbol{r} \tag{4}$$

where $\mu = GM$, with G being the Universal Gravitational constant, r is the position vector from M to m. Radial Equation of Motion for a particle is given by

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \tag{5}$$

which can be further be solved supposing $r = u^{-1}$where u = u(θ) and θ = θ(t). It follows that

$$u(\theta) = \frac{GM}{h^2}[1 + ecos(\theta - \theta_o)] \tag{6}$$

where e and θ_o are arbitrary constants. Without loss of generality, we can set $\theta_o = 0$ by rotating our coordinate system about the z-axis. We define $r_c = \frac{h^2}{GM}$ here and can also assume that $e \geq 0$. Thus, we obtain,

$$r(\theta) = \frac{r_c}{1+ecos\theta} \tag{7}$$

This is the equation of a conic section that is confocal with the origin (i.e., with the Sun). Here specifically, for e < 1, Equation (7) is the equation of an ellipse, for e = 1, it is the equation of a parabola and finally for e > 1, Equation (7) is the equation of a hyperbola [8]. We can also write

$$r = \frac{a(1-e^2)}{1+ecos\theta} \tag{8}$$

where a is the semi major axis or mean orbital radius and e is the orbital eccentricity.

5.2 Perturbed Problem

The fundamental problem of celestial mechanics involves determining how an orbit, which would naturally follow a conic section if unperturbed, is altered by a small disturbing force $\boldsymbol{dF}$ that is minor compared to the central force $\boldsymbol{F}$. The orbital elements that define the osculating orbit are known as the osculating elements. Once these elements are expressed as functions of time, the position of the particle can be determined at any given moment. The primary objective is to ascertain the time rate of change of the set $(a, e, i, \omega, \Omega, \tau)$ due to the influence of the small disturbing force (Burn et al., 1976) [5].

The disturbing force $\boldsymbol{dF}$ can be decomposed into its components along an orthogonal unit vector triad $(\hat{e}_r, \hat{e}_T, \hat{e}_N)$, where $\hat{e}_r$ is the radial unit vector, directed outward along the position vector $\boldsymbol{r}, \hat{e}_T$, is the transverse unit vector, perpendicular to $\hat{e}_r$ in the orbital plane, and $\hat{e}_N$ is the normal unit vector, perpendicular to the orbital plane and directed in the direction of $\hat{e}_r \times \hat{e}_T$. Consequently, the perturbing force $\boldsymbol{dF}$ can be expressed as $\boldsymbol{dF} = R\hat{e}_r + T\hat{e}_T + N\hat{e}_N$ where R is the radial component, T is the transverse component, and N is the normal component of the perturbing force [9]. So, perturbation equation for semi-major axis and eccentricity are given as follows,

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}[eRsinf + T(1 + ecosf)] \qquad (9)$$

$$\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}[Rsinf + T(cosf + cosE_{ecc})] \qquad (10)$$

where E_{ecc} is eccentric anomaly given as,

$$tan\frac{E_{ecc}}{2} = \sqrt{\frac{1-e}{1+e}}tan\frac{f}{2}$$, where 'f' is true anomaly.

6. Results & Analysis

The Figure 2 shows the relation between semi-major axis perturbations with varying heliocentric distance. So, as we go from Mercury to Mars the semi-major axis perturbation gradually decreases and perturbation flattened towards the sun's side that could be due to the influence of net effect of different forces on IDP's.
Similarly Figure 3 shows eccentricity perturbation with heliocentric distance. And we observed that the perturbation in eccentricity compared to the semi-major axis perturbation is much faster.

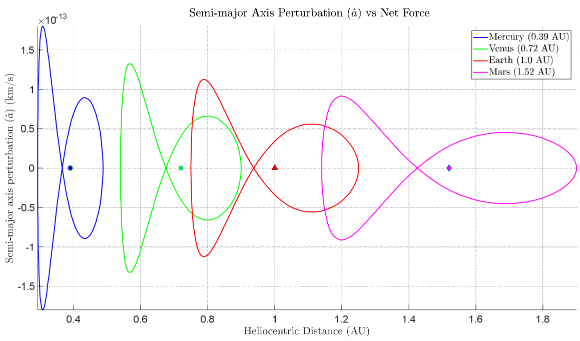

Figure 2. Semi-Major axis perturbation Vs. Heliocentric Distance; Mercury, Venus, Earth & Mars are represented as Blue Circle, Green Square, Red Triangle, Pink Diamond respectively.

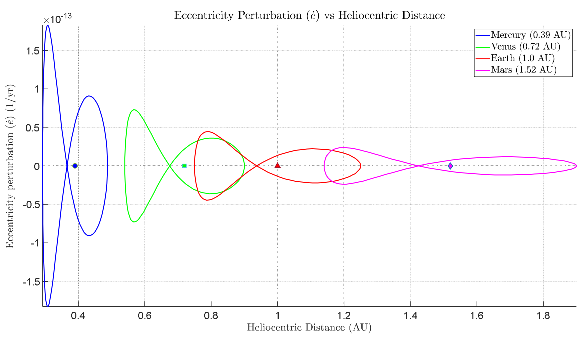

Figure 3. Eccentricity perturbation Vs. Heliocentric Distance

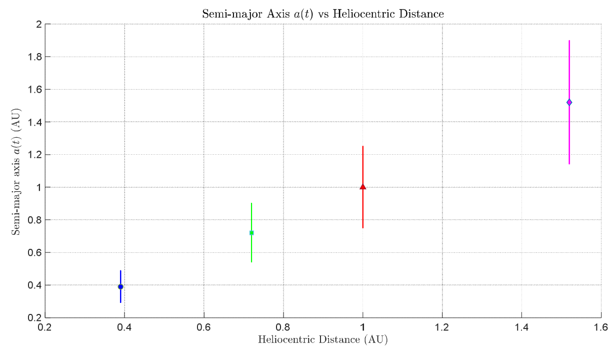

Figure 4. Semi-Major Axis Vs. Heliocentric Distance

Here Figure 4 shows the range of semi-major axis of the dust particle with respect to Sun around the planets. And from Figure 5 we concluded that eccentricity decreases as we go from Mercury to Mars.

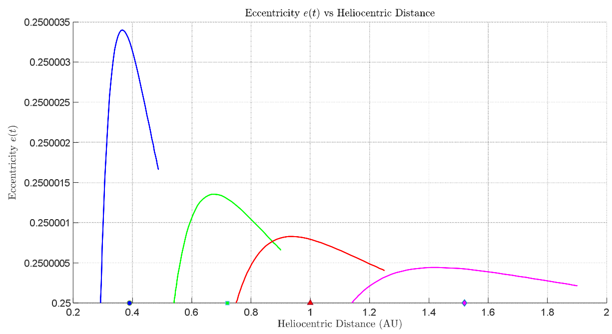

Figure 5. Eccentricity Vs. Heliocentric Distance

7. Summary and Conclusion

When the Interplanetary Dust start its journey from various places like from outer solar system, Kuiper belt & mostly from the Asteroid belt and during its journey the dust is affected by the various Gravitational and Non-Gravitational forces which may lead to change its path towards the Sun. And during their journey towards the Sun mostly the dust is trapped around orbit of the planets and this dust trapping by the planets may lead to the formation of fainted dust rings.

The path of dust around the planet is mostly influenced by the various forces like gravitational force of sun and planets and majorly influenced by the Solar Wind Force and Solar Radiation Force So, the net effect of these forces may lead the change in their orbital parameters (semi-major axis, eccentricity, inclination etc.).

And the net effect of the forces on the dust is useful to understand dynamical evolution of Interplanetary dust in the Solar System.

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7. References

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