Polarized EiBI-gravitating dust molecular cloud stability with bispectral electron distribution

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Abstract

A partially ionized dust molecular cloud (DMC), consisting of both neutral and charged dust grains, along with the usual plasma components, considerably modifies the traditional Jeans instability. This leads to the development of a novel Jeans-like hybrid mode, termed as the pulsational mode of gravitational collapse (PMGC). A semi-analytic DMC model is developed to explore the collective effects of the dust-polarization force and Eddington-inspired Born-Infeld (EiBI) gravity on the PMGC in the presence of (r,q)-distributed electrons. The ions are treated with the Maxwellian distribution law (thermal), while the electrons are thermostatistically treated with the bispectral (r,q)-distribution law (non-thermal). The partially ionized dust grains are modelled as their respective constitutive fluids distinctly. A unique quartic form of the linear PMGC dispersion relation is derived using the spherical normal mode analysis over the perturbed DMC. The oscillatory and propagatory characteristics of the high-frequency mode are investigated in a judicious numerical platform. It shows that an increase in the polarization force and negative EiBI parameter substantially enhances the instability growth, causing the DMC to undergo self-gravitational collapse and vice versa. The outcomes reported here could be utilized in understanding the bounded structure formation processes in various real astrophysical scenarios.

1 Introduction

The Jeans instability is a canonical mechanism responsible for the self-gravitational fragmentation of large, complex gaseous molecular clouds, leading to the formation of bound structures within galaxies [1]. However, the ubiquitous presence of interstellar dust has been observed in star-forming regions of the molecular clouds in the mid-infrared (MIR) and far-infrared (FIR) bands [2]. These dust grains, ranging from micron to sub-micron sizes, are primarily composed of graphite, silicate, and other complex carbon derivatives. The self-gravitating molecular clouds filled with electrons, ions, and dust particles are often referred to as dust molecular clouds (DMCs). The dust grains within DMCs can become electrically charged through various processes such as contact electrification, secondary electron emission, thermionic emission, etc. [3]. The charged dust present in the DMCs generates a repulsive electrostatic force that acts against the inward gravitational force. Additionally, due to the higher mass of the dust grains compared to the lighter plasma components (electrons and ions), the self-gravitational effects of the dust particles significantly modifies the traditional Jeans instability criteria for dusty interstellar clouds [4]. In a partially ionized DMC, where both charged and neutral dust particles are present, a new Jeans-like mode, known as the pulsational mode of gravitational collapse (PMGC), emerges [5]. This hybrid mode results from the interplay between the electrostatic force (acting as a deforming force) of the charged dust grains and the self-gravitational force (acting as a restoring force) of the collective constituents.

In the last two decades, the PMGC research area have extensively been explored in both linear and non-linear regime [3, 6]. Additionally, factors like turbulence [7], viscoelasticity [8], ion drag [9], and non-thermality [6] have been considered in the studies of PMGC dynamics. Furthermore, effects of modified gravity have also been incorporated in the PMGC research area by several authors [10, 4]. However, the effects of polarization force, a critical aspect of nonuniform plasmas (with uneven electron and ion distribution), has yet to be extensively explored in the PMGC instability dynamics. In inhomogeneous plasmas with negatively charged dust grains, the Debye sheath is formed by ions and the deformation of these plasma sheaths results in the development of polarization force. A simplified expression for the polarization force of negatively charged dusty systems can be given as $\vec{F}_p = -Z_d e R (n_i/n_{i0})^{1/2} \vec{\nabla}\phi$ (see [4] for details). Here, $R = Z_d e^2/(4T_i \lambda_{Di0})$ is called the polarization interaction parameter, determining the impact of the polarization force.

Recent observations have unveiled that the space plasma particles often do not follow the Maxwellian velocity distribution, rather their velocity distribution shows features like high energy tails and flat-tops [11]. Space plasmas often process a number of superthermal (non-thermal) electrons, and to model them a number of velocity distribution have been proposed such as the kappa distribution, bispectral $(r,q)-$distribution, Cairns distribution, etc. Among all the velocity distribution functions, the non-thermal (r,q)-distribution can model both the flat-tops and high energy tails simultaneously. Hence, (r,q)-distribution provides more flexibility over other non-thermal velocity distribution functions.

Despite of the remarkable success of Einstein's theory of general relativity (GR), it is not free from limitations, such as the cosmic expansion, formation of singularities, dark energy and dark matter, etc. To resolve these issues, many extensions of GR in the form of modified Einstein-Hilbert action have been proposed. The Eddington inspired Born-Infeld (EiBI) gravity is one of such modifications of GR introduced in 2010 [12]. Within the EiBI gravity regime, the traditional gravitational Poisson equation gets modified in the non-relativistic limit, which is now used for phenomenological studies in bounded astrophysical self-gravitating structures.

Looking into the significant roles of the EiBI gravity, (r,q)-distributed electrons (non-thermal), and polarization force in the stability of a DMC, a generalized semi-analytic model is proposed to study the collective effects of the aforementioned factors in the PMGC stability dynamics.

After the introduction in section 1, the physical model setup and the basic governing equations used to derive the linear dispersion relation (quartic in degree) are outlined in section 2. In section 3, the numerical outcomes, along with figures, are discussed. Finally, the results are concluded in section 4.

2 Basic model and dispersion relation

A weakly coupled, nonuniform, and globally quasi-neutral self-gravitating DMC is considered to study the collective effects of polarization force and EiBI gravity. It consists of four species, namely, the non-thermal (r,q)-distributed electrons, the thermal Maxwellian ions, neutral, and negatively charged dust grains. Due to the lower ionization level $\sim 10^{-7}$ [5], both charged and neutral dust grains along with electrons and ions are found in such DMCs. A spherically symmetric approximation (radial 1-D case) is considered rather than a full spherical geometry (spherical 3-D case) as the DMCs are usually confined in spherical volumes. This simplification holds good when the fluctuation wavelength significantly exceeds the grain-to-grain distance [6].

The electrons are assumed to follow the bispectral (r,q) distribution [13], whereas, the ions are considered to follow the Maxwellian velocity distribution on a slow dust inertial time scale [6]. The spherically symmetric hydrodynamic equations for the DMC can be written with all usual notations as

$$\frac{\partial}{\partial t}n_{dn}+\rho^{-2}\frac{\partial}{\partial \rho}\left(\rho^{2}n_{dn}v_{dn}\right)=0, \tag{1}$$

$$\frac{\partial}{\partial t}v_{dn}+(v_{dn}\frac{\partial}{\partial \rho})v_{dn}=-\frac{\partial}{\partial \rho}\psi, \tag{2}$$

$$\frac{\partial}{\partial t}n_{dc}+\rho^{-2}\frac{\partial}{\partial \rho}\left(\rho^{2}n_{dc}v_{dc}\right)=0, \tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial t}v_{dc}+\left(v_{dc}\frac{\partial}{\partial \rho}\right)v_{dc}=\frac{Z_d e}{m_d}\frac{\partial}{\partial \rho}\phi-\frac{Z_d e}{m_d}R\left(\frac{n_i}{n_{i0}}\right)^{\frac{1}{2}}\\ \frac{\partial}{\partial \rho}\phi-\frac{\partial}{\partial \rho}\psi-v_{cn}\left(v_{dc}-v_{dn}\right),\end{aligned} \tag{4}$$

$$\rho^{-2}\frac{\partial}{\partial \rho}\left(\rho^{2}\frac{\partial}{\partial \rho}\phi\right)=4\pi e\left(n_e-n_i+Z_d n_{dc}\right), \tag{5}$$

$$\begin{aligned}\rho^{-2}\frac{\partial}{\partial \rho}\left(\rho^{2}\frac{\partial}{\partial \rho}\psi\right)=4\pi G m_d\left(n_{dc}+n_{dn}-n_{d0}\right)+\\ \left(\frac{\chi}{4}\right)\rho^{-2}\frac{\partial}{\partial \rho}\left(\rho^{2}\frac{\partial}{\partial \rho}\left[m_d(n_{dc}+n_{dn}-n_{d0})\right]\right).\end{aligned} \tag{6}$$

Here, ϕ and ψ represent the electrostatic and gravitational potential. χ serves as the EiBI parameter that has been constrained for various astrophysical objects, such as white dwarfs ($\chi \leq 4.86\times10^{3}\ \mathrm{kg^{-1}\,m^{5}\,s^{-2}}$), neutron stars ($\chi < 10^{-2}\ \mathrm{kg^{-1}\,m^{5}\,s^{-2}}$), Sun ($\chi \leq 3\times10^{5}\ \mathrm{kg^{-1}\,m^{5}\,s^{-2}}$), etc [4]. Furthermore, $v_{cn(nc)}$ is accountable for the collisional effects. It is noteworthy that eq. (6) reduces to usual Poisson equation for $\chi=0$. Additionally, the planar equations can be recovered for $\rho\to\infty$, ρ being the radial distance.

Application of the standard perturbation followed by the spherical Fourier analysis and the method of substitution, decoupling, and elimination [10] on the eqs. (1)-(6) yields in a unique quartic dispersion relation (normalized)

$$\Omega^{4}+a_{3}\Omega^{3}+a_{2}\Omega^{2}+a_{1}\Omega+a_{0}=0; \tag{7}$$

$$a_{3}=i\left(\frac{v_{cn}}{\omega_J}\right), \tag{8}$$

$$\begin{aligned}a_{2}=\left(1-\frac{\chi m_d n_{d0}}{4C_{DA}^{2}}K^{2}-\frac{(1-R)\omega_J^{2}\lambda_{D_{r,q}}^{2}\Omega_{pdc}^{2}}{K^{2}\omega_J^{2}\lambda_{D_{r,q}}^{2}+C_{DA}^{2}}K^{2}\right)\times\\ \left(K^{2}+\frac{1}{\xi^{2}}\right)K^{-2},\end{aligned} \tag{9}$$

$$a_{1}=i\left(\frac{v_{cn}}{\omega_J}\right)\left[\left(1-\frac{\chi m_d n_{d0}}{4C_{DA}^{2}}K^{2}\right)\left(K^{2}+\frac{1}{\xi^{2}}\right)K^{-2}\right], \tag{10}$$

$$\begin{aligned}a_{0}=-(1-R)\Omega_{pdc}^{2}\left(\frac{\lambda_{D_{r,q}}}{\lambda_J}\right)^{2}\frac{C_{DA}^{2}}{(K^{2}\omega_J^{2}\lambda_{D_{r,q}}^{2}+C_{DA}^{2})}\times\\ \left(1-\Omega_{Jc}^{2}-\frac{\chi K^{2}}{4C_{DA}^{2}}m_d n_{dn0}\right)\left(K^{2}+\frac{1}{\xi^{2}}\right)^{2}K^{-2}.\end{aligned} \tag{11}$$

Here, $\omega_{Jd}=(4\pi G m_d n_{d0})^{1/2}$, $\omega_{Jcd}=(4\pi G m_d n_{dc0})^{1/2}$, $\omega_{pd}=\left(4\pi q_d^{2}n_{d0}/m_d\right)^{1/2}$, and $\omega_{pdc}=\left(4\pi q_d^{2}n_{dc0}/m_d\right)^{1/2}$ are the effective critical Jeans frequency for the dust species, critical Jeans frequency for the charged dust particles, effective dust-plasma oscillation frequency, and charged dust-plasma oscillation frequency, respectively. Furthermore, $\xi=\rho/\lambda_{Jd}$ and $K=k(C_{DA}/\omega_{Jd})$ are the Jeans-scaled radial distance and angular wavenumber normalized with the Jeans length (λ_{Jd}) and angular Jeans wavenumber ($k_{Jd}\equiv\omega_{Jd}/C_{DA}$, C_{DA} being the dust acoustic phase speed), respectively. Additionally, $\Omega=\omega/\omega_{Jd}$, $\Omega_{pdc}=\omega_{pdc}/\omega_{Jd}$, and $\Omega_{Jc}=\omega_{Jc}/\omega_{Jd}$ are the Jeans-scaled angular frequency, charged dust-plasma oscillation frequency, and critical Jeans frequency for charged dust normalized with the Jeans angular frequency (ω_{Jd}). The term $\lambda_{D_{r,q}}=\left[(T_eT_i)/(4\pi e^{2}(An_{e0}T_i+n_{i0}T_e))\right]^{1/2}$ is the effective dust-plasma Debye length for (r,q)-distribution [4].

To study the high-frequency stability dynamics of the complex self-gravitating DMC the normalized dispersion relation (7) is transformed into a high-frequency (HF) regime as

$$\Omega^2 + i\left(\frac{\nu_{cn}}{\omega_J}\right)\Omega + \left(1 - \frac{\chi m_d n_{d0}}{4C_{DA}^2}K^2 - \frac{(1-R)\lambda_{D_{r,q}}^2\omega_{pdc}^2}{K^2\omega_J^2\lambda_{D_{r,q}}^2 + C_{DA}^2}K^2\right) \times \left(K^2 + \frac{1}{\xi^2}\right)K^{-2} = 0. \tag{12}$$

Equation (12) is the required quadratic dispersion relation for studying the high-frequency PMGC stability of the DMC.

3 Results and discussions

The derived quadratic polynomial dispersion relation (12) is used for numerical analysis, and the resulting graphical representations (figures 1-2) are respectively interpreted. The various adopted equilibrium values of different parameters are collected from the literature [5] as $m_e = 9.1 \times 10^{-28}$ g, $m_i = 1.6 \times 10^{-24}$ g, $m_d = 4 \times 10^{-12}$ g, $e = 4.80 \times 10^{-10}$ esu, $q_d = -100e$, $r_d = 1.5 \times 10^{-10}$ cm, $n_{e0} = 1.2 \times 10^6$ cm^{-3}, $n_{i0} = 4.95 \times 10^6$ cm^{-3}, $n_{dc0} = 2.35$ cm^{-3}, $n_{dn0} = 4$ cm^{-3}, $T_e = 1$ eV, $T_i = 0.8$ eV, and $T_d = 0.01$ eV. Using these parametric values, the Coulomb coupling parameter Γ_{Cou} is calculated as ~ 0.167, the Jeans angular frequency (ω_J) is estimated as 4.61×10^{-9} rad s^{-1}, the dust thermal velocity (V_{td}) is found as 1.4×10^{-1} cm s^{-1}, and the dust-plasma oscillation frequency (ω_{pd}) is determined as 0.21 rad s^{-1}.

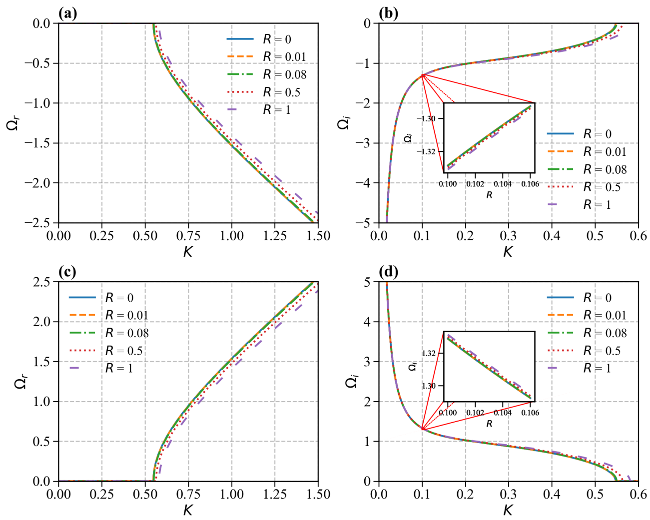

Figure 1: Variation of the real frequency (Ω_r) and growth rate (Ω_i) of the 1st (a and b) and the 2nd root (c and d) of the dispersion relation with the angular wavenumber (K) for different values of the polarization interaction parameter (R) on the Jeans scale. The various curves represents: (a) $R = 0$ (blue solid line), (b) $R = 0.01$ (yellow dashed line), (c) $R = 0.08$ (green dashed-dotted line), (d) $R = 0.5$ (red dotted line), and (e) $R = 1$ (purple loosely dashed line), respectively.

Figure 1 depicts the effect of the polarization force on the PMGC instability in the presence of (r,q)-distributed electrons in the EiBI gravity framework. Here, the Jeans-scaled real frequency (Ω_r) and growth rate (Ω_i) are plotted against the Jeans-scaled angular wavenumber (K) for different values of polarization parameter ($R = 0, 0.01, 0.08, 0.5$, and 1). The EiBI parameter is chosen as $\chi = 2 \times 10^8$ g^{-1} cm^5 s^{-2}, the normalized radial distance as $\xi = 10$, and the multi-order non-thermality coefficient as $A = 1.4$ (for $r = 0$ and $q = 5$). It is clear from figure 1 that the 1st root of the dispersion relation is a stable one, whereas, the 2nd root is unstable. Hence, to study the PMGC instability, the 2nd root is used hereafter. Figure 1d demonstrates that the growth of the instability is more significant for longer wavelengths (gravitation-like), i.e., small K-values. Furthermore, it is observed in figure 1 that an increase in the value of R increases the growth rate of the PMGC instability and decreases the propagtion of the real frequency. This leads to a less stable system (most stable for $R = 0$ and least stable for $R = 1$) which indicates that in the EiBI gravity framework, the polarization force acts as a destabilizing agent for negatively charged dust. It should be noted here that for the considered weakly coupled DMC, the polarization interaction parameter is calculated to be $R \sim 5.9 \times 10^{-20}$, which is $\ll 1$. Therefore, the polarization force is not expected to significantly affect the stability of a weakly coupled system.

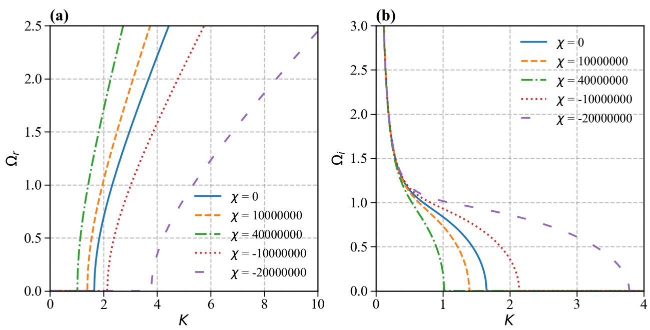

Figure 2: Same as figure 1, but for the 2nd root and different values of the EiBI parameter (χ). The various curves respectively link to: (a) $\chi = 0$ (blue solid line, Newtonian reference line), (b) $\chi = 1 \times 10^8$ g^{-1} cm^5 s^{-2} (yellow dashed line), (c) $\chi = 4 \times 10^8$ g^{-1} cm^5 s^{-2} (green dashed-dotted line), (d) $\chi = -1 \times 10^8$ g^{-1} cm^5 s^{-2} (red dotted line), and (e) $\chi = -2 \times 10^8$ g^{-1} cm^5 s^{-2} (purple loosely dashed line).

Figure 2 shows influence of the EiBI gravity on the PMGC instability in the presence of dust-polarization force and (r,q)-distributed electrons. For a self-gravitating astrophysical object of size L, the value of χ should be less than GL^2, with G representing the universal gravitational constant [14]. For the DMC under consideration having a modified critical Jeans length, $\lambda_{Jd} \sim 1.37 \times 10^8$ cm, the constraint is calculated as $\chi < 1.33 \times 10^9$ g^{-1} cm^5 s^{-2}. The χ-value is varied in the range from -2×10^8 g^{-1} cm^5 s^{-2} to 4×10^8 g^{-1} cm^5 s^{-2}. It is important to note here that, $\chi = 0$ serves as the Newtonian reference in the non-local gravity, which is in fact, a crucial feature for assessing the observed trends. All the other parameters are kept the same as mentioned at the beginning of this section, except $R = 0.01$ and $A = 1.4$ $(r = 0, q = 5)$. It is clear from figure 2 that an increase in the positive EiBI parameter (χ) reduces the growth rate of the instability, leading to a greater stability. However, an increase in the negative EiBI parameter (χ) observed to drive the system into higher instability in comparison to those in the Newtonian scenarios. Therefore, it can be deduced that the positive EiBI parameter behaves as a stabilizing agent, whereas the negative EiBI parameter acts as a destabilizing agent.

4 Conclusions

A semi-analytic model formalism is proposed to study the combined influence of the EiBI gravity, (r,q)-distributed electrons, and dust-polarization force on the pulsational mode of gravitational collapse (PMGC) in complex dust molecular clouds (DMCs). Application of a standard spherical wave analysis yields a generalized quartic dispersion relation. The derived dispersion relation fairly corroborates with the previous results reported in the literature [5] in the absence of newly added sophistications. The dispersion relation is then numerically illustrated with relevant parametric inputs to explore different PMGC stability features. The PMGC instability is found to be significantly more pronounced for longer wavelengths (gravitational-like) than the shorter wavelengths (acoustic-like). Additionally, the instability shows a saturating propensity for higher K-values. It is seen that the polarization force and the negative EiBI parameter act as destabilizing factors, while the positive EiBI parameter acts in favour of the DMC stability. Besides, it is concluded that the polarization force is not a significant factor for the stability of a weakly coupled system but may have greater importance for strongly coupled systems. The outcomes of the PMGC instability analyses reported here can be utilized to conceive diverse active processes responsible for astrophysical structure formation, particularly in the H II regions (ionized hydrogen regions, plasmic) of the DMCs, such as Sh2-87, Sh2-88B, Sh2-235, and so forth. It, however, has fundamental restrictions for applicability in the H I regions (non-ionized hydrogen regions, non-plasmic) of the star-forming DMCs.

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