



Instability behaviors of bipolar dusty plasma fluids in Saturnian mid-magnetosphere

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Abstract

This study investigates the instability behaviors of bipolar dust grains in the Saturn's middle magnetosphere (SMM), in the vicinity of the moon Enceladus. The system investigated herein includes bipolar dust grains treated as fluids; Maxwellian cold electrons and ions (primarily, protons, H^+ ; water-group ions, W^+); and suprathermal electrons. The dynamics of the charged dust grains incorporates the pressure gradient force, Lorentz force, and ion and neutral drag forces. A fluctuation in charged grains is expected as we move radially outward, which is also included in this model. A standard cylindrical wave mode analysis results in a generalized monic linear sextic dispersion relation. The multi-parametric influences on the relative growth rate variational features are illustratively investigated. The instability propagation frequency and the corresponding phase velocity of the resultant normal acoustic wave mode are found to lie well within the range of dust-acoustic wave modes propagating in similar space plasma environs as previously reported in the literature.

1. Introduction

Saturn's E-ring is known as the largest planetary ring of the solar system that has a radial extension from around $2.5 R_S$, R_S being the radius of Saturn ~ 60000 km, up to $20 R_S$. The magnetosphere of Saturn can be classified in three regions: the inner magnetosphere, the middle magnetosphere, and the outer magnetosphere, depending on the variability of their composition and magnetic field configuration [1]. The Saturn's middle magnetosphere (SMM) begins just outside the orbit of Enceladus and covers most of the E-ring. Enceladus is the sixth largest moon of Saturn and is believed to have water ocean beneath its icy surface. It is geologically very active and feeds materials to the diffuse E-ring in the form of icy grains from its south pole [2]. The Cassini Plasma Spectrometer (CAPS), the Radio and Plasma Wave Science (RPWS) instrument inbuilt in the Cassini have investigated the composition of plasma as well as the plasma parameters derived from the charged particle distributions [3-5]. Density models have been developed by power-law fits on the data measurements provided by these instruments. The large dispersion in the observational spectra suggests that the plasma is highly variable in the region between the orbits of Enceladus and Tethys ($3 - 5 R_S$) [6]. Both Voyager 1 and 2 observed cold

electron component ($1 - 100$) eV and hot electron component ($1 - 100$) keV [7]. However, the CAPS detects lower energies compared to that observed by Voyager missions. According to the CAPS measurements, low energy electrons (≤ 10 eV) exist within the radial extension of $\sim 6.3 R_S$. However, the hotter component of electrons having energy ≥ 100 eV exist most likely due to the rapid transport via the centrifugal interchange instability (CII) [4]. Another important instrument, the Cosmic Dust Analyzer (CDA), built specifically for the detection of dust, have found silicate to be the most dominant composition of the μm -sized dust grains [5]. The dust grain surface potential is expected to be highly variable in the vicinity of Enceladus orbit due to the variation in the plasma compositions which eventually affects the dust charging mechanisms.

The investigation of dust dynamics in the Saturnian magnetosphere has been an interesting field in the dusty plasma domain. The Saturnian rings serve as best possible natural laboratory to investigate various dusty plasma phenomena and numerous investigations have already been reported [8-10]. In this work we are going to study the instability behaviors of bipolar dusty fluids in the vicinity of Enceladus. The drag force is responsible for the distribution of dust grains throughout the E-ring which initially generated from the Enceladus [11], and therefore drag forces can be considered to play a significant role in the dynamics of dust grains. Apart from the drag forces, dynamics of both the dust grains incorporates the pressure gradient force and the Lorentz force. The model also includes Maxwellian cold electrons and ions, and suprathermal electrons. The variable surface potential could also play an important role in dust grain dynamics and hence it is incorporated in the model via the dust charging equation. A standard cylindrical wave analysis yields a generalized monic linear sextic dispersion relation whose propagatory features indicates that the instability propagates as dust-acoustic wave. The multiparametric influences on the relative growth rate features is illustrated.

The model formalism and derived dispersion relation, following the brief introduction in Section 1, are elaborated in Section 2. Section 3 covers the results and discussion. Finally, Section 4 presents the conclusion of this work along with its relevant astrophysical applications.

2. Model formalism

In this work, we intend to study the instability behaviors of bipolar dusty fluids in Saturn's middle magnetosphere, in the vicinity of the moon Enceladus. Usually, dust charging mechanism is dominated by the flow of plasma current making the dust surface potential negative, however in space and astrophysical environments dust grains may acquire positive charge due to the photoemission processes [12]. The theoretical model considered in this work consists of bipolar dust grains, treated as fluids. The other plasma constituents include Maxwellian cold electrons and ions, and suprathermal electrons are described with the kappa distribution. The governing usual fluid model equations are formulated in axisymmetric cylindrical coordinate system. The dynamics of the dust grains are mainly governed by the pressure gradient force, Lorentz force, and neutral and ion drag forces. In the Saturn's E-ring, the collisional ion drag is found to dominate over scattering phenomena [13]. Therefore, the ion drag force due to scattering is justifiably ignored.

After a standard process of linear normal mode analysis followed by substitution, elimination, and simplification, one finally obtains a generalized monic linear sextic dispersion relation given in usual symbolism as

$$\Omega^6 + A_1\Omega^5 + A_2\Omega^4 + A_3\Omega^3 + A_4\Omega^2 + A_5\Omega + A_6 = 0 \quad (1)$$

The various multiparametric coefficients in the expanded form are given below

$$A_1 = i[a_1(n_{dn0}/n_{i0}) - \alpha_1(\Gamma_2 + K^2 + 1/4R^2)][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (2)$$

$$A_2 = [-\alpha_2(\Gamma_2 + K^2 + 1/4R^2) + (n_{dn0}/n_{i0})(a_3Z_{dn0} - a_1a_7) - \Gamma_3][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (3)$$

$$A_3 = [-\alpha_7Z_{dn0}(n_{dn0}/n_{i0}) + i\{-\alpha_3(\Gamma_2 + K^2 + 1/4R^2) - (n_{dn0}/n_{i0})(a_1a_8 + Z_{dn0}\alpha_8) - \Gamma_3(a_2 + a_4)\}][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (4)$$

$$A_4 = [-\alpha_4(\Gamma_2 + K^2 + 1/4R^2) - (n_{dn0}/n_{i0})(a_1a_9 + Z_{dn0}\alpha_9) + \Gamma_3(a_2a_4 + a_5) - i\alpha_{10}Z_{dn0}(n_{dn0}/n_{i0})][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (5)$$

$$A_5 = [-\alpha_{11}Z_{dn0}(n_{dn0}/n_{i0}) + i\{-\alpha_5(\Gamma_2 + K^2 + 1/4R^2) - (n_{dn0}/n_{i0})(a_1a_{10} + Z_{dn0}\alpha_{12}) + \Gamma_3(-a_6 + a_2a_5)\}][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (6)$$

$$A_6 = [-\alpha_6(\Gamma_2 + K^2 + 1/4R^2) - (n_{dn0}/n_{i0})(a_1a_{11} + Z_{dn0}\alpha_{13}) + \Gamma_3a_2a_6 - i\alpha_{14}Z_{dn0}(n_{dn0}/n_{i0})][\Gamma_2 + K^2 + 1/4R^2]^{-1} \quad (7)$$

Here, $R = r/\lambda_D$ is the normalized radial coordinate with dusty plasma Debye length as the normalizing parameter given as $\lambda_D = (k_B T_i / 4\pi n_{i0} e^2)^{1/2}$. The time coordinate, $T = t\omega_{pd}$ is normalized by the relaxation time for dust

particles given as $\omega_{pd} = (4\pi n_{dn0} Z_{dn}^2 e^2 / m_{dn})^{1/2}$. K is the normalized angular wavenumber. The other small coefficients (a 's and α 's) depend on the various plasma parameters relevant to the system under consideration.

3. Results and discussions

The derived sextic monic dispersion relation is solved numerically to study the instability behaviors of the considered system. The values of the various parameters are obtained from in-situ observations by Cassini in the vicinity of the orbit of Enceladus as reported in the literatures [3-6]. In Figure 1, we show the relative growth rate compared to the propagation characteristics (Ω_i/Ω_r) with variation in the normalized angular wavenumber for different equilibrium cold electron density (n_{ec0}). The relative growth rate is found to decrease with increase in the cold electron density indicating the stabilizing nature of n_{ec0} . Figure 2 shows the same as Figure 1, but for different values of negative dust-to-ion charge density ratio ($H_{dn} = Z_{dn0}n_{dn0}/n_{i0}$). It is seen that the relative growth rate increases with increase in the value of H_{dn} . It may be attributed to the fact that the increase in the negative dust-to-ion charge density ratio will enhance the collisional frequency between the negative dust grains and ions making the system unstable.

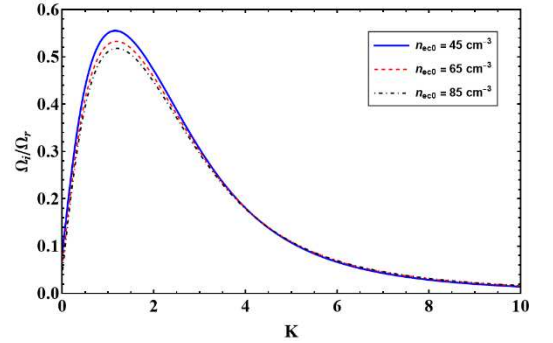


Figure 1. Profile of the normalized relative growth rate (Ω_i/Ω_r) with variation in the normalized angular wavenumber (K) but for different values of the equilibrium cold electron density (n_{ec0}).

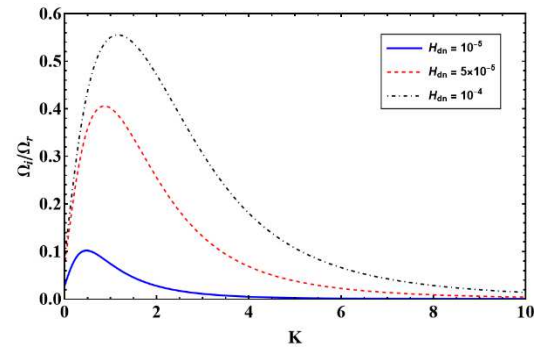


Figure 2. Same as Figure 1, but for different values of the equilibrium negative dust-to-ion charge density ratio ($H_{dn} = Z_{dn0}n_{dn0}/n_{i0}$).

Figure 3 depicts the same as Figure 1 but for different values of the positive dust-to-ion charge density ratio ($H_{dp} = Z_{dp0}n_{dp0}/n_{i0}$). It is seen that with increase in the positive dust charge density the relative growth rate decreases making the system stable. Moreover, in Figure 4, we show the influence of various ionic species, that are present in the Saturnian magnetosphere, on the relative instability growth rate. It is seen that the water group ions (W^+) have greater impact on the instability growth as compared to the hydrogen ions (H^+). It is noteworthy to mention here that the instability considered herein has propagation frequency $\sim 10^{-2}$ s $^{-1}$ and a phase velocity of value ~ 0.1 cm s $^{-1}$. It reliably matches well with the dust-acoustic wave modes propagating in the space plasma environments as previously reported in the literature [14].

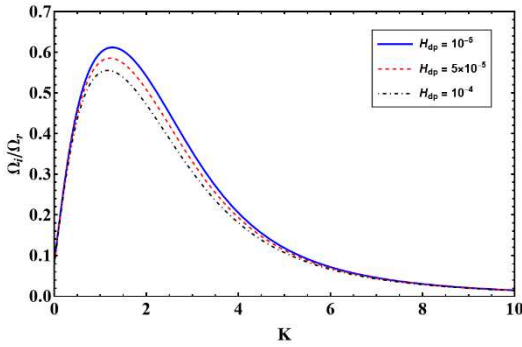


Figure 3. Same as Figure 1, but for different values of the equilibrium positive dust-to-ion charge density ratio ($H_{dp} = Z_{dp0}n_{dp0}/n_{i0}$).

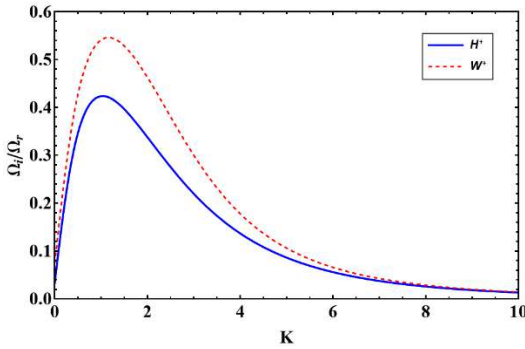


Figure 4. Same as Figure 1, but for different ionic species: Hydrogen ion (H^+) and water group ions (W^+).

4. Conclusions

In this work, the instability behaviors of bipolar dusty plasma fluids have been investigated in Saturnian mid-magnetosphere (SMM) near the orbit of Enceladus. A standard cylindrical wave analysis yields a generalized monic linear sextic dispersion relation revealing wave propagatory features like usual dust-acoustic wave modes. The multiparametric influences on the relative growth rate variational features is illustrated (Figures (1)-(4)). The cold electron number density (n_{ec0}) and the positive dust-to-ion charge density ratio (H_{dp}) are found to stabilize the plasma

system, while the negative dust-to-ion charge density ratio (H_{dn}) makes the system unstable. The presence of the water-group ions in the SMM makes the system unstable due to increase in the collisions with the negatively charged dust grains. The instability propagatory features, such as the frequency, phase velocity, etc., investigated herein fairly lie in the range of previous dust-acoustic wave modes. This presented theoretic analysis could be extensively useful in understanding the diverse observed collective acoustic wave phenomena in the complex Saturnian magnetosphere as reported elsewhere [15, 16].

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7. References

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