Nonthermal pulsational mode stability

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Abstract

We present a semi-analytic model formalism to study the stability properties of the pulsational mode of gravitational collapse (PMGC). It is triggered in a strongly coupled, superextensive, and spherically symmetric dust molecular cloud (DMC). It includes the active roles of viscosity, nonthermality, and negative ionic effects. The spatial behavior of the high-energetic lighter compositions is described by the nonthermal kappa (κ)-distribution law. The heavier cold dust grains are treated in the viscous fluid fabric. Application of the spherical normal mode treatment reduces the DMC to a generalized linear quartic dispersion relation. A computational modeling tool enables us to see the diverse growth features. It is interestingly found that certain physical parameters, such as the dust temperature, negative ionic temperature, and dust size, play the stabilizing roles on the PMGC mode against self-gravity. The results investigated here could be useful in conceiving astrophysical structure formation processes in the DMCs.

1. Introduction

The dust molecular clouds (DMCs) are often regarded as stellar nurseries for diverse astrophysical bounded structure formation mechanisms in the interstellar media (ISM) [1-3]. It is relevant to mention that a considerable portion of the DMCs is mainly composed of the highly massive, highly charged, and micron-sized dust grains. In a broader sense, such astrostructure formation mechanisms are inherently intricate phenomena, supported by various types of collective waves and instabilities phenomena [4-5]. One such type of collective fluctuation phenomenon is termed as the pulsational mode of gravitational collapse (PMGC), which was initially reported by Dwivedi et al. in 1999 for describing the star formation processes [6]. The PMGC mode arises as a result of the dynamic coupling of the two non-overlapping force fields, the non-electrostatic self-gravity and the non-gravitational electric field [6-15]. The PMGC is founded on a gravito-electrostatic interplay triggering astrostructure formation process [6].

In the preceding, a good number of researchers has been extensively explored the stability problem of the PMGC mode stability in a self-gravitating, unbounded DMC, particularly within the planar geometry framework [6-15]. The reported analyses outlined in the literature have comprehensively investigated the fluctuation dynamics in both the linear [6-7, 11-13, 15] and non-linear [8-10, 14] regimes. Subsequent studies are available that highlight the crucial roles of the diversified astrophysical factors such as the dust charge fluctuations [7, 10, 13-14], viscoelasticity [11, 14-15], nonthermal effects [13, 15], etc. The proposed work provides a comprehensive analysis founded on the ongoing research of the same phenomenon under diverse novel astronomical circumstances.

It has been reported in the literature that the presence of the negative ionic species plays a significant role in initiating non-local gravitational collapse dynamics responsible for bounded structure formation processes [16-17]. The existence of the various negative ions in self-gravitating dusty plasmas causes a significant change in the dust-charging processes amid the local quasi-neutrality conditions [16-17]. Thus, after being inspired by the above well-established scenarios, we develop a theoretical formalism to study the PMGC stability dynamics in a spherically symmetric DMC. It includes the important realistic factors, here afresh, such as nonthermality effects, viscosity effects, negative ionic effects, and so forth. This study with the constitutive negative ions contributes a useful element to support the observational evidences revealing the star formation process in the H_{II} regions, which has been detected extensively with the help of the JWST and other space-based observations [18].

Apart from the brief introduction in Section 1, the model formalism and the dispersion relation are described in Section 2. Section 3 portrays the results and discussions properly. Section 4 finally outlines a concise summary of our semi-analytical analysis.

2. Model Formalism

We adopt a simplified theoretical model to study the PMGC mode stability dynamics in a spherically symmetric, self-gravitating, and strongly coupled ($\Gamma_{\mathrm{Cou}}\sim 99 \gg 1$) DMC. The multi-constitutive DMC is mainly composed of lighter nonthermal electrons, positive and negative ions; and inertial neutral and charged dust grains. The considered spherical model is presumed to exist in a quasi-neutral, hydrostatic, and homogeneous equilibrium configuration. By adopting the spherically symmetric geometry (radial direction only), we simplify the complex

3-D model to a 1-D model configuration [19]. This approach inherently excludes the polar and azimuthal aspects for the sake of analytic simplicity. The inclusion of the negative ions is basically grounded on the fact that it plays a crucial role in the bounded structure formation mechanisms through gravitational collapse dynamics in diversified astrophysical circumstances [16-17]. The suprathermal species are assumed to satisfy the thermostatistically anti-equilibrium κ-distribution law [19]. The heavier dust grains are considered as the viscous fluids. The dust charge fluctuation dynamics is not considered here as $\tau_d/\tau_{ch} \sim 10^6 \gg 1$, in accordance with the prior findings [20].

The collisional effects of all the nonthermal plasma species with the massive unipolar dust grains are excluded to simplify our proposed analysis founded in a generalized hydrodynamic (GH) model fabric. The dynamics of the viscous fluid composed of the neutral and charged dust grains in the spherical co-ordinate (r, t) is portrayed by the continuity equations, momentum equations, and the closure electro-gravitational Poisson equations. Analytic operation over these equations in accordance with the Fourier-formulaic analysis and collective parametric decoupling approach results in a linear quartic unnormalized dispersion relation. Then, the adoption of a standard astrophysical normalization scheme [15] yields in a normalized linear quartic dispersion relation with an atypical set of multiparametric dispersion coefficients can be respectively expressed as

$$\Omega^4 + A_3\Omega^3 + A_2\Omega^2 + A_1\Omega^1 + A_0\Omega^0 = 0 \quad (1),$$

$$A_3 = i[F_{cn} + K^2(\eta^*_{dn} + \eta^*_{dc})] \quad (2),$$

$$A_2 = [(K^2 + \xi^{-2})K^{-2}\{1 - \Omega^2_{Pd}K^2\Lambda^2_{D\kappa}(K^2\Lambda^2_{D\kappa} + 1)^{-1}\} - K^2\eta^*_{dn}(F_{cn} + K^2\eta^*_{dc})] \quad (3),$$

$$A_1 = i(K^2 + \xi^{-2})[F_{cn}K^{-2} + \eta^*_{dc}\{1 + \omega^2_{Jdc}\omega^{-2}_{Jd}(\eta^*_{dn}/\eta^*_{dc} - 1)\} - \{\Omega^2_{Pd}K^2\Lambda^2_{D\kappa}\eta^*_{dn}(K^2\Lambda^2_{D\kappa} + 1)^{-1}\}] \quad (4),$$

$$A_0 = -[\Omega^2_{Pd}\Lambda^2_{D\kappa}(K^2 + \xi^{-2})^2(1 - \omega^2_{Jdc}\omega^{-2}_{Jd})][K^2(K^2\Lambda^2_{D\kappa} + 1)]^{-1} \quad (5).$$

Here we are interested in the Ω_i-features only. As a consequence, Equation (1) is simplified to yield the PMGC growth or damping rate in the kinetic limits [11], which accounts for the high-frequency regime cast as

$$\Omega_i = [-A_3 + (A_3^2 + 4A_2)^{1/2}]/2, \quad (6).$$

Here, $\xi = r/\lambda_{Jd}$, $K = k/k_{Jd}$, and $\Omega = \omega/\omega_{Jd}$ denote the Jeans-rescaled radial distance, angular wavenumber, and angular frequency, respectively. The other normalized notations are $F_{cn} = \nu_{cn}/\omega_{Jd}$ for charged-neutral dust collisional frequency, $\Omega_{Pd} = \omega_{Pd}/\omega_{Jd}$ for dust-plasma oscillation frequency, and $\Lambda_{D\kappa} = \lambda_{D\kappa}/\lambda_{Jd}$ for adapted Debye length thermostatistically tailored by the conjoint action of the κ-distributed species. Here, $\nu_{cn} = \pi a^2 n_{dn0} v_{td}$ represents the said charged-neutral dust collisional frequency with πa^2 corresponds to the angular cross-section and $v_{td} = (k_B T_d/m_d)^{1/2}$ is the thermal speed of the isothermal identical dust grains, each with radius a. The terms λ_{Jd}, ω_{Jd}, and k_{Jd} are the Jeans length, Jeans frequency, and Jeans critical wavenumber associated with the dust fluid, respectively. Additionally, $\eta^*_{dn} = \eta_{dn}/\eta_{Jn}$ and $\eta^*_{dc} = \eta_{dc}/\eta_{Jc}$ denote the Jeans-rescaled viscosities of the neutral and charged dust fluids, respectively. The Jeans viscosities associated with the neutral and charged dust grains are given respectively as $\eta_{Jn} = m_d n_{dn0}\omega^{-1}_{Jd}c^2_{SS}$ and $\eta_{Jc} = m_d n_{dc0}\omega^{-1}_{Jd}c^2_{SS}$.

3. Results and discussions

The PMGC mode undergoes decay or growth in DMCs, depending appreciably on its diversified parametric properties. After the above analytic-based study, we proceed numerically to explore the exact nature of the diverse dispersive features of the collective instability excitation. The different input values of relevant physical parameters are adopted from diverse resources seen in the literature [8-12, 19]. The numerical analysis, highlighting microphysical characteristics and insightful aspects of the collective eigenmode is demonstrated and displayed.

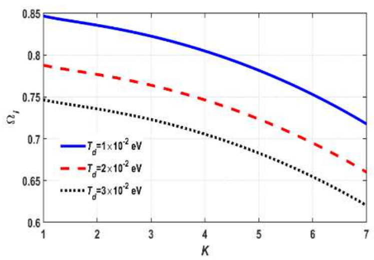

Figure 1. Profile of the Jeans-rescaled PMGC growth rate (Ω_i) with the variation in the Jeans-rescaled angular wavenumber (K) for different T_d-values.

As it is shown in Figure 1, we depict the Jeans-rescaled PMGC growth rate (Ω_i) with variation in the Jeans-rescaled angular wavenumber (K) corresponding to different dust temperature (T_d) values. The profile features three distinct curves reflecting three different spatial cases: (a) Solid blue line is for $T_d = 1 \times 10^{-2}$ eV, (b) Dashed red line for $T_d = 2 \times 10^{-2}$ eV, and (c) Dotted black line for $T_d = 3 \times 10^{-2}$ eV. It is clear that the magnitude of the instability growth rate decreases as the T_d value increases, and vice-versa. It indicates that T_d exhibits a tendency to stabilize the perturbed fluctuations against the inward non-local self-gravity. Physically, this occurs as a consequence of the fact that an enhancement in the T_d-value directly results in an increment in the non-gravitational thermal pressure associated with the dust grains. As a result, the competency of the gravitational pressure support amidst the dust grains reduces. Thereby, it triggers a greater stability configuration by opposing the cloud collapse dynamics.

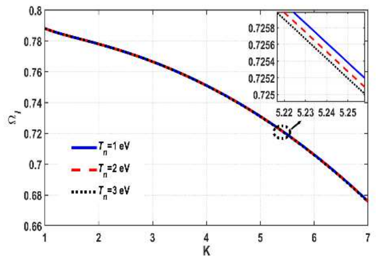

Figure 2. Same as Figure 1, but for different T_n-values.

In Figure 2, we display the similar profile but for distinct specified values of the negative ionic temperature (T_n). It displays three distinct curves reflecting three different conditions: (a) Solid blue line is for $T_n = 1$ eV, (b) Dashed red line for $T_n = 2$ eV, and (c) Dotted black line for $T_n = 3$ eV. To enhance further clarity, the main figure is supplemented by the respective zoomed-in profiles of the relevant sections. It is revealed that, with a rise in the T_n-value, the profile structures show similar behaviors as Figure 1, and vice-versa. It may be attributable to the frictional coupling mechanisms of the dust grains with the constitutive negative ions. It hereby implies that, the higher T_n-values causes an increase in the thermal motion of the negative ions. This may lead to more frequent and energetic collisions with the dust grains. Moreover, it impacts the momentum transfer rate and increases negative ionic-driven current flow to the dust surface. Accordingly, the electrostatic Coulomb repulsive force among the like-charged dust grains increases. As a corollary, it impedes the strength of the self-gravitational interaction dynamics and leads to system stability. Thus, T_n acts as a stabilizing factor on the collective PMGC mode.

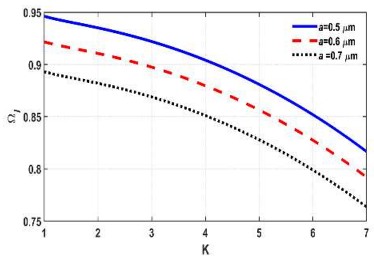

Figure 3. Same as Figure 1, but for different a-values.

As in Figure 3, we analogously represent the similar profiles as Figure 1, but now with variation in the dust grain size (a) values. It displays three distinct curves reflecting three different conditions: (a) Solid blue line is for $a = 0.5$ μm, (b) Dashed red line for $a = 0.6$ μm, and (c) Dotted black line for $a = 0.7$ μm. We herewith see that, with a boost in the a-value, the profile patterns show similar structure as in Figure 1, and vice-versa. It is important to highlight that an enlarged size of the dust grain provides more potential sites for electrons to interact with. In addition to this, the larger-sized dust grains tend to have a lower photoemission rate. This, in turn, implies a net accumulation of electrons on the dust surface and tends to increase the propensity of the Coulombic repulsive force among the dust grains. Subsequently, the considered system acquires optimal stability against the inward gravitational forces. Thus, it is pertinent to mention that, a, plays a stabilizing role on the collective fluctuations of the PMGC origin against its inward self-gravity.

It is quite likely that our stability analyses are widely useful in understanding the gravito-electrostatically bounded astrostructure formation processes in diverse realistic astrophysical environments, as largely realizable in H_{II} regions, including RCW 38, RCW 36, etc. [19, 21]. It is also important to mention that our analyses provide theoretical supports to many observational evidences detected via JWST and various other space-based astronomical observations [18].

4. Conclusions

A semi-analytic theoretic model is reported to study the pulsational mode of gravitational collapse dynamics (PMGC) excitable in a spherical dust molecular cloud (DMC). The considered model is composed of nonthermal electrons, positive and negative ions, neutral and charged massive dust grains, etc. The nonthermal species (lighter) are assumed to follow the κ-distribution law within a non-gravitational framework. However, the heavier cold dust grains (neutral and charged) exhibit the viscous fluid-like behavior. The generalized hydrodynamic (GH) model fabric, systematically closed by the electro-gravitational Poisson formalism, captures a complete description of the dust-dust dynamical behavior. The non-planar normal mode decomposition over the closed DMC against its local equilibrium leads to a generalized linear quartic dispersion relation. It contains a distinct set of dispersion coefficients governing the PMGC dynamics in diverse astronomical conditions. A numerical analysis is accomplished to showcase the PMGC behaviors in detail within realistic astrophysical scenarios on a common graphical platform. It is mainly seen that an augmentation of the dust temperature increases the PMGC growth rate, and vice-versa (Figure 1). In addition, it is also found that, with an increase in the negative ionic temperature (Figure 2) and the dust size (Figure 3), the DMC behaves progressively more resistant to such gravitational instabilities, and vice-versa.

At last, it is worth mentioning here that the proposed analysis has its own merits and demerits admittedly. The presented work lays the foundation for further refinement of the physical model to understand such astrophysical phenomena in a more realistic way. This would require the proper inclusion of diverse asymmetric factors, such as the zeroth-order gradient effects, dust Brownian motions, large-scale inhomogeneities, tidal force field effects, azimuthal and polar geometric contributions, and so forth.

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