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# Development of Compressive Sensing based SAR Image Reconstruction using $\ell_1$ −norm Coordinate Decent Optimization

Saurav Ganguly* (1), and Mainak Mukhopadhyay (2)
(1) Presidency University, Bengaluru 560064, India, https://presidencyuniversity.in/
(2) Birla Institute of Technology MESRA, Ranchi 835215, India

## Abstract

Synthetic Aperture Radar (SAR) imaging plays a crucial role in remote sensing applications, offering high-resolution images for various environmental monitoring and disaster management tasks. Traditional SAR imaging systems typically produce large volumes of data, leading to significant challenges in data storage, transmission, and processing. The Compressive Sensing (CS) approach, developed few years ago, has emerged as a promising framework, to address these challenges by enabling the reconstruction of high-quality SAR images from highly under-sampled data. This paper proposes a compressive sensing (CS) compression scheme adapt for high dynamic range synthetic aperture radar (SAR) images. The scheme utilizes the $\ell_1$ norm-coordinate decent optimization for efficiently reconstructing compressed data. Experimental results show that the classification of the reconstructed images by proposed method bring out better outcome in terms of peak signal-to-noise ratio (PSNR), mean-square-error (MSE) and relative error.

## 1. Introduction

A distinctive radar detection technique, popularly known as synthetic aperture radar (SAR) imaging, is capable of creating high-resolution pictures of the landscape and stationary surface targets [1 –3]. Generally, SAR systems are placed on moving platforms, such as spacecraft or aircraft, to synthesize larger aperture, so as to obtain high resolution in the azimuth direction. Furthermore, wide bandwidth signals are used to generate high resolution in the range direction. The technique used to collect SAR data requires unambiguous processing of data to generate an image that can be realized by the human perception [4 – 6]. The collected raw data have to either be stored onboard, or sent to a receiver station using a specific data connection link. As a result, compression of the raw data becomes a necessity and a beneficial choice for designing SAR systems. Traditional methods used for SAR data compression incline to be more computationally complex and substantially error prone in reconstruction.

A recently developed signal processing and reconstruction technique, known as compressive sensing (CS), has revolutionized data acquisition by capturing sparse signals using significantly fewer samples than traditional methods [7]. By exploiting the sparsity, inherent in many natural signals, compressive sensing enables efficient reconstruction of the original signal from sparse measurements [7–10]. This approach has found applications in various fields, such as medical imaging, radar, and communication systems, where reducing data acquisition and transmission costs without compromising signal quality is crucial [11–17]. SAR images often exhibit high effectual spectrum due to varying reflectivity of terrain and objects, necessitating robust compression techniques to preserve essential details while reducing data volume. Compressive sensing (CS) offers a promising solution by exploiting the sparse nature of SAR images in transform domains.

In this paper, the proposed method of $\ell_1$ norm-coordinate decent minimization, enhances the reconstruction accuracy by leveraging the convex optimization framework to recover sparse signals effectively. The proposed scheme aims to achieve significant compression ratios while maintaining reconstruction fidelity suitable for high-quality SAR image applications. Experimental results in terms of variation of peak signal-to-noise ratio (PSNR), mean-square-error (MSE) and relative error, demonstrate the effectiveness of the proposed approach in compressing and reconstructing high dynamic range SAR images, showcasing its potential for practical deployment in remote sensing and surveillance systems.

## 2. Basic Characteristics of SAR Image Data

Synthetic Aperture Radar (SAR) data is obtained by transmitting RF signals towards the Earth's surface (in the direction of the region where an image is needed) from an aircraft or satellite, as shown in fig. 1. The radar antenna receives the signals that are reflected back from the surface, capturing both the amplitude and phase of the returned signals. By moving the radar antenna along a path (either on the aircraft/satellite or electronically synthesized), SAR synthesizes a large antenna aperture, allowing for high-resolution imaging and the creation of detailed radar images of the Earth's surface.

SAR image formation involves processing the collected data. This process utilizes the phase history data $S(t, \theta)$ collected by the radar antenna as it moves along a path $\theta$. By applying the Fourier transform along the range dimension (distance from radar to target) and then the inverse Fourier transform along the azimuth dimension (perpendicular to the flight path), the SAR image $I\ (p, q)$ is formed, which is given as [18–20]:

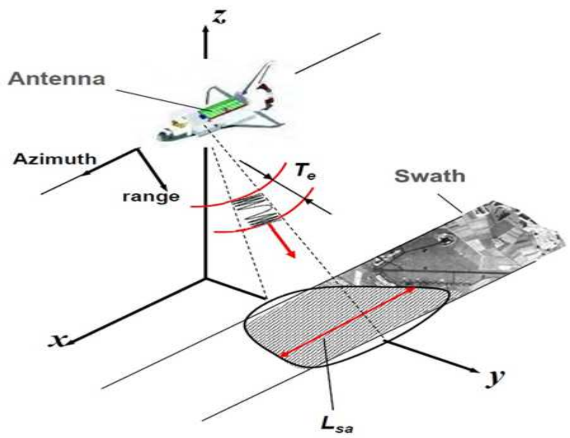


Fig. 1. A scheme of data acquisition system employed by SAR

$$I(p, q) = \mathcal{F}^{-1}\{\mathcal{F}\{S(t, \theta)\}\} \quad (1)$$

Here, $p$ and $q$ denote the spatial coordinates in the image plane, $t$ represents time or range, and $\theta$ represents the angle or azimuth position of the antenna. This processing synthesizes a large antenna aperture, achieving high-resolution imaging of the Earth's surface from the radar data.

Fourier transformation and back projection, although quiet simple to implement possess some significant drawbacks. High processing complexity is a major disadvantage. Also effect of speckle noise, limited temporal resolution and difficulty in handling non-uniform sampling lead to distortions, and hence degrade the reconstructed image quality.

## 3. SAR Data Acquisition by Compressive Sensing

Compressive sensing theory states that if a discrete signal or image is sparse in some basis, it can be precisely reconstructed with fewer samples than the Nyquist rate, using a vector, $f = \mathbf{\Psi}x$, where $\boldsymbol{x} \in \mathbb{R}^N$, is a vector having few non-zero elements in it, $k \ll N$, and $\mathbf{\Psi} = \mathbb{R}^{N \times N}$, is a matrix with basis vectors in its columns [7, 8, 10–12]. In this framework, the image or signal is obtained using linear projections of, $y = \mathbf{\Phi}f$ , where $y$ is the measurement vector, such that $y \in \mathbb{R}^M$ , and $\mathbf{\Phi}$ is the measurement matrix, given as $\mathbf{\Phi} = \mathbb{R}^{M \times N}$. Now considering $x$ (which is $k$ −sparse) in the in the measurement vector, we get:

$$y = \mathbf{\Phi\Psi}x \Rightarrow y = \mathbf{\Theta}x \quad (2)$$

where $\mathbf{\Theta} \in \mathbb{R}^{M \times N}$, is the observation matrix [7, 8,12]. Here, $N$ represents the length of the SAR image, and $M$ is the number of measurements.

The sparsity basis matrix, $\mathbf{\Psi}$ typically represents the dictionary of possible reflectivity patterns or responses that the SAR image can exhibit. Designing of a stable measurement (or sensing) matrix $\mathbf{\Phi}$, and its incoherency with $\mathbf{\Psi}$, is one of the most important factors to attain, for effective realization of sparse sampling and reconstruction. This incoherency can be achieved by the Restricted Isometric Property (RIP) of SAR imaging in the sparse domain [8–10].

Designing the sensing matrix $\mathbf{\Phi}$ involves creating a matrix that efficiently samples the radar reflections from the scene. Here are three key considerations:

1. Spatial Sampling: The sensing matrix should account for the spatial resolution required in the SAR image. This typically involves placing virtual radar sensors in a grid pattern to ensure adequate spatial coverage.
2. Signal-to-Noise Ratio (SNR): To optimize the quality of measurements, the sensing matrix should consider the SNR characteristics of the radar system. This might involve strategic placement of radar pulses and coherent processing to maximize SNR.
3. Waveform Design: The waveform used in SAR plays a crucial role in determining the characteristics of the sensing matrix. The choice of waveform affects aspects like resolution, range ambiguity, and Doppler effects, which in turn influence the design of the sensing matrix.

To recover or reconstruct back the $k$ −sparse vector $x$, it is required that the number of measurements, $M$, must be greater than $k$. Evidently, $M$ would be quite less than $N$ (underdetermined system), i.e., $k < M \ll N$ . Several mathematical models and methodologies are developed over the years to recover $x$ efficiently, suited for specific applications [8 –17]. Algorithms like Orthogonal Matching Pursuit (OMP) [11–13] and Iterative Hard Thresholding (IHT), iteratively select and refine sparse representations or estimate signals directly from underdetermined systems with high efficiency and accuracy, fairly similar to $\ell_0$ −norm regularization. Other methods, such as Basis Pursuit (BP) [8] find the sparsest representation by minimizing the $\ell_1$ −norm of the coefficients subject to data consistency constraints.

### 3.1. $\ell_1$− norm based coordinate decent minimization for compressed SAR signal reconstruction

Coordinate Descent is a method that iteratively updates each coefficient of the solution vector $x$, by minimizing the objective function with respect to one coefficient while fixing all others. It is particularly efficient for $\ell_1$ − regularization due to its ability to handle the nonsmooth nature of the $\ell_1$ penalty. $\ell_1$− regularized coordinate decent substitute a sum of absolute values for the sum of squares used in $\ell_2$ − regularization [7, 8], to obtain:

$$minimize \|\mathbf{\Phi}x - y\|_2^2 + \lambda\|x\|_1 \quad (3)$$

where $\lambda$ is a variable of normalization, such that $\lambda > 0$. Compressive sampling seeks to make use of the compressibility or sparsity characteristics of appropriate SAR image signal in the transform domain by solving a minimization problem of the form:

$$minimize\|\mathbf{\Phi}V - y\|_2^2 + \lambda\|UV\|_1 \quad (4)$$

where $\mathbf{\Phi} = [\Phi_1, \Phi_2 \dots \dots \Phi_M]^T \in \mathbb{R}^{M\times N}$ is the compressed sensing or measurement matrix. $V \in \mathbb{R}^N$ is a variable and $U$ is the sparsity transform, such that $\Phi U^{-1} \in \mathbb{R}^{M\times N}$. Eq.(4) is the methodology used for compressed SAR image reconstruction at the earth/ground station.

## 4. Simulation Results

Taking into consideration two cases to evaluate the effectiveness of the suggested algorithm using the compressive sensing paradigm. The SAR image is considered of a ground target or region [18–20].

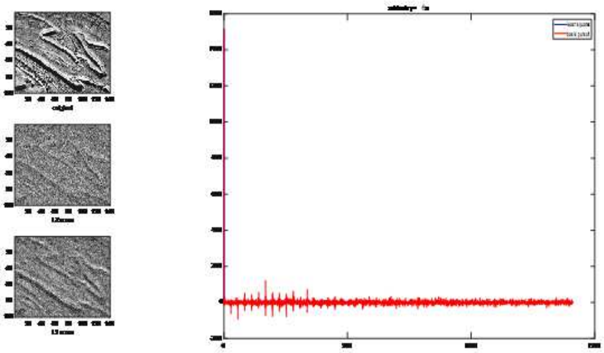

Fig. 2. SAR image recovery (left column) using 10% and sparsity by compressive sensing (Top: the original image Middle: $\ell_2$ –norm recovery, Bottom $\ell_1$ –norm recovery with the proposed algorithm)

Fig 2 above shows a comparison of simulated output of the reconstruction of SAR image by the proposed methodology and a standard $\ell_2$ – regularization based algorithm [8]. Examples of test images following reconstruction using the suggested method at 10% subsampling ratios are depicted in fig. 3.

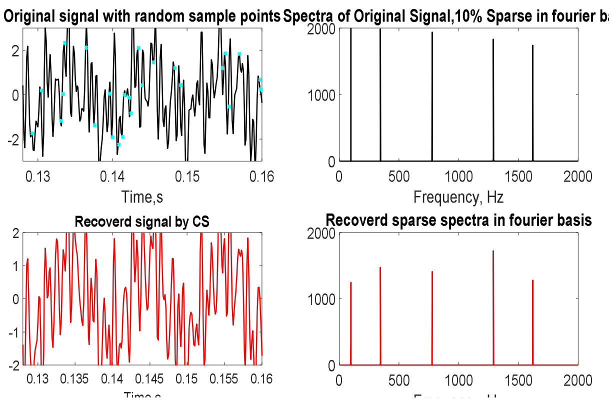


Fig.3. The simulated Image after the reconstruction with the proposed methodology at 10% sub-sampling ratios

In evaluate the performance of the proposed method in SAR image reconstruction, the Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), and Relative Error (RE) [7–17] measurements are computed. The MSE, (PSNR, and RE measurements between the original and reconstructed pictures using the suggested approaches for various subsampling ratios are calculated in Table 1.

Table-1: Simulation of Performance Evaluation

| Compression Ratio | MSE | PSNR (dB) | RE |
|---|---|---|---|
| 10 % | 0.0654 | 70.127 | 0.0047 |
| 30 % | 0.7893 | 62.991 | 0.0188 |
| 50% | 4.367 | 56.126 | 0.0511 |
| 75% | 60.154 | 43.124 | 0.1776 |
| 90% | 130.24 | 40.887 | 0.2574 |

When the compression is lower (between 10% and 50% missing samples), the suggested methodology performs flawlessly in reconstructing the original SAR image. When the compression is moderate (between 50% and 75% missing samples), it performs acceptably. When the compression is higher (more than 75% missing samples), the original signal or image is not well recovered.

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